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Computer-Aided Analysis of Dissipation Losses in Isolated and Coupled Transmission Lines for Microwave and Millimeter-Wave Integrated-Circuit Applications

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The analysis employs a quasi-TEM model for uniform isolated transmission lines and for the even- and odd-mode transmission lines associated with coupled-line structures. The conductor and dielectric losses in these structures are then related to equivalent charge-density distributions, which are evaluated using a method-of-moments solution. The transmission lines treated by this analysis may contain any number of lossy conductors and inhomogeneous dielectrics, consisting of any number of different homogeneous dielectric regions. After explicit expressions are developed for use in evaluating conductor and dielectric loss coefficients for the even and odd modes of

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20. Abstract (Continued)

coupled transmission lines a development is provided to explicitly relate the four-port, terminal, electrical performance of directional couplers to the modal loss coefficients.

Losses are evaluated for examples of four isolated transmission lines and one coupled transmission line. For microstrip and coplanar waveguide the computed loss coefficients are in reasonable agreement with experimental data. For inverted microstrip and trapped inverted microstrip, evaluations presented in both tables and graphs provide useful design information for circuit applications. A comparison is made of the total loss characteristics of microstrip, coplanar waveguide, inverted microstrip, and trapped inverted microstrip. The utility of the analysis for coupled-transmission-line losses is illustrated for the example of edge-coupled microstrip with a dielectric overlay by comparing computed loss characteristics with measured values. The accuracy of the loss evaluations is quantitatively assessed, and suggestions are made for additional refinements in the solutions.

The five computer programs employed to evaluate dissipation losses for microstrip, coplanar waveguide, inverted microstrip, trapped inverted microstrip, and edge-coupled microstrip with a dielectric overlay are listed. For each computer program information is provided as to storage requirement, execution time, compatibility with various commercial computer systems, and input/output data descriptions.

CONTENTS

COMPUTER-AIDED ANALYSIS OF DISSIPATION LOSSES IN ISOLATED AND COUPLED TRANSMISSION LINES FOR MICROWAVE AND MILLIMETER-WAVE INTEGRATED-CIRCUIT APPLICATIONS

INTRODUCTION

There is considerable interest in investigating and exploiting new transmission lines for use in integrated circuits operating at higher microwave and millimeter-wave frequencies. This interest has been spurred by the success in reducing circuit cost, size, and weight through the application of microstrip at lower to intermediate microwave frequencies. Unfortunately microstrip is discouragingly lossy and more difficult to fabricate at higher microwave and millimeter-wave frequencies. These considerations have prompted the search for transmission lines that are amenable to integrated-circuit fabrication methods (thin film and photolithographic technology) and that have better loss characteristics than microstrip.

To facilitate the investigation of transmission lines which offer potential for improvements over microstrip at the frequencies of interest, a flexible computer-aided analysis of transmission-line losses has been implemented. This analysis is suitable for application to a wide variety of transmission lines. This report is a description of the implementation of that analysis as it applies to both isolated and coupled transmission lines, where losses due to both conductor and dielectric dissipation are taken into account. Various examples of loss evaluations using the analysis are presented for both isolated and coupled transmission lines of interest. For the examples of isolated microstrip and coplanar waveguide [1] calculated loss characteristics are compared to experimentally determined loss parameters, thereby enabling an assessment of the accuracy of the analysis. These examples are followed by a presentation of computed loss characteristics for inverted microstrip [2] and trapped inverted microstrip. Also presented are evaluations of loss parameters for edge-coupled microstrip with a dielectric overlay. Calculated and experimentally determined loss characteristics are compared. Explanations are given for the use of computer programs listed in Appendixes A through E which are useful for computing loss coefficients due to conductor and dielectric dissipation for microstrip, coplanar waveguide, inverted microstrip, trapped inverted microstrip, and edge-coupled microstrip with a dielectric overlay.

FORMULATION OF ANALYSIS FOR EVALUATION OF LOSSES

The following analysis of conductor-loss and dielectric-loss characteristics for isolated and coupled uniform transmission lines is consistent with the quasi-TEM models described in Refs. 3 and 4. For isolated transmission lines the direction of propagation is taken to be along the z direction. Consistent with the quasi-TEM model and the transmission-line wave-approach described in Ref. 5, the z dependence for voltage and current along the transmission line is accounted for by a factor $e^{-\gamma z}$, where

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$$\gamma = \alpha + j\beta, \tag{1}$$

in which α is the attenuation constant due to conductor and dielectric losses and β is the phase constant. Following the development set forth in Ref. 5, the attenuation constant α is given by

$$\alpha = \frac{\bar{P}_d}{2\bar{P}_f}, \tag{2}$$

where $\bar{P}_d$ is the time-averaged power dissipated per unit length and $\bar{P}_f$ is the time-averaged power flow along the line.

In the following subsection explicit expressions are developed for the quantities $\bar{P}_d$ and $\bar{P}_f$. It is these expressions which were used to provide the results presented in the third section after appropriate incorporation within the computer programs, described in the fifth section and listed in Appendixes A through E.

Isolated Transmission Lines

To obtain a useful expression for $\bar{P}_f$ in Eq. (2), the following treatment is employed. By virtue of Eq. (1) a $+z$-traveling wave on the transmission line is of the form

$$V = V_0 e^{-(\alpha+j\beta)z}, \quad I = \frac{V}{Z_0} \ . \tag{3}$$

The complex power flow is given by

$$P_f = VI^* = \frac{|V_0|^2}{Z_0^*} \ e^{-2\alpha z} , \tag{4}$$

where I^* and Z_0^* are the conjugates of I and the characteristic impedance respectively. Then $\bar{P}_f$ is given by

$$\bar{P}_f = (Re\ Z_0) \ \frac{|V_0|^2}{|Z_0|^2} \ e^{-2\alpha z} \ . \tag{5}$$

As is shown in Appendix F, for the type of transmission lines considered here, $Re\ Z_0$ and $|Z_0|$ are nearly equal to $(Z_0)_{LL}$, the characteristic impedance of the lossless line. By virtue of this consideration $\bar{P}_f$ can be expressed as

$$\bar{P}_f = |V_0|^2 \ vCe^{-2\alpha z} , \tag{6}$$

where v is the phase velocity and C is the electrostatic capacitance per unit length.

In the next two subsections of this report explicit expressions are developed for evaluating isolated-transmission-line conductor-loss and dielectric-loss coefficients α_c and α_d, respectively. The total-loss coefficient α is obtained from these by summing α_c and α_d.

Conductor Losses

To obtain an expression which is useful for evaluating $\bar{P}_d$ in Eq. (2) for losses due to imperfect conductors, the approximation described in Ref. 6 is employed. $\bar{P}_d$ can be expressed approximately by

$$\bar{P}_{d,c} \approx \int_{\substack{\text{conductor}\\ \text{surfaces}}} |H_0|^2 R \, d\ell, \tag{7}$$

where the additional subscript c on $\bar{P}_{d,c}$ denotes power losses due to imperfect conductors, $|H_0|^2$ is the amplitude squared of the magnetic field at conducting surfaces for the lossless case, and R is the surface resistance of the metals in the system. For good conductors R can be written as

$$R = \sqrt{\frac{\pi f \mu_0}{\sigma}}\,, \tag{8}$$

where f is the frequency of the propagating wave, μ_0 is the free-space permeability, and σ is the conductor's dc conductivity.

Consistent with the quasi-TEM-propagation assumption for transmission media inhomogeneously filled with dielectric,

$$\mathbf{H}_0 \approx \frac{1}{\eta_{\text{eff}}} \mathbf{u}_z \times \mathbf{E}_0\,, \tag{9}$$

where η_{eff} is the effective intrinsic impedance, $\mathbf{u}_z$ is the unit vector in the z direction, and $\mathbf{E}_0$ is the electric field for the lossless case at points just outside the conductor surfaces. By virtue of Eq. (9)

$$|H_0|^2 \approx \frac{|E_0|^2}{\eta_{\text{eff}}^2} = \frac{\epsilon_{\text{eff}}}{\mu_0} |E_0|^2\,, \tag{10}$$

where ϵ_{eff} is the effective permittivity, determined as described in Ref. 3. Using the results of Eqs. (10) and (8) in Eq. (7), $\bar{P}_{d,c}$ can be written as

$$\overline{P_{d,c}} \approx \frac{\epsilon_{\text{eff}}}{\mu_0} \sqrt{\frac{\pi f \mu_0}{\sigma}} \int_{\substack{\text{conductor}\\ \text{surfaces}}} |E_0|^2 \, d\ell \cdot \tag{11}$$

In the lossless TEM solutions described in Refs. 3 and 4, the electric field at a point along the surface of a perfect conductor is given by

$$E_0 = 2\pi q, \tag{12}$$

where q is the equivalent charge density residing at the conductor surface (sum of free and polarization charge densities). In Ref. 3 the free-charge-density distribution along the surface of the N_jth conductor, in a system having N_c conductor surfaces, is approximated by a pulse expansion as

$$q^{N_j} = \sum_{i=1}^{(N_S)_j} q_i^{N_j} P_i^{N_i}(i), \; N_j = 1, \dots, N_c. \tag{13}$$

This representation arises by subdividing the contour defined by the surface of the N_jth conductor into $(N_S)_j$ segments. Along the ith segment the free-charge-density distribution is taken to be constant at the value $q_i^{N_j}$. $P(i)$ is a pulse function defined by

$$P^{N_j}(i) \begin{cases} = 1 \text{ on the } i\text{th section of } N_j & \text{(14a)} \\ = 0 \text{ on all other sections of } N_j. & \text{(14b)} \end{cases}$$

Then $\bar{P}_{d,c}$ in Eq. (11) can be rewritten, using Eqs. (12) and (13),

$$\bar{P}_{d,c} \approx \frac{4\pi^2 \epsilon_{\text{eff}}}{\mu_0} \sqrt{\frac{\pi f \mu_0}{\sigma}} \sum_{j=1}^{N_c} \sum_{i=1}^{(N_s)_j} \left(q_i^{N_j}\right)^2 \Delta\ell_i^{N_j}, \tag{15}$$

where $\Delta\ell_i^{N_j}$ is the length of the ith segment on the N_jth conductor.

Finally the loss coefficient due to conductor losses α_c can be written using Eqs. (15), (6), and (2) as

$$\alpha_c \approx \frac{\frac{4\pi^2 \epsilon_{\text{eff}}}{\mu_0} \sqrt{\frac{\pi f \mu_0}{\sigma}}}{2\,|V_0|^2 v\, C} \sum_{j=1}^{N_c} \sum_{i=1}^{(N_s)_j} \left(q_i^{N_j}\right)^2 \Delta\ell_i^{N_j} \text{ (neper/unit length)}. \tag{16}$$

It is the expression given in Eq. (16) which is embodied in the computer programs described later in this report. This expression has been used to provide the design information for conductor losses presented later in the report. To obtain the coefficients α_c (in dB per unit length) the following relationship is provided for completeness

$$\alpha_c \text{ (dB/ unit length)} = \frac{20}{\ln 10}\ \alpha_c \text{ (neper/unit length).} \tag{17}$$

Dielectric Losses

To obtain an expression which is useful for evaluating $\bar{P}_d$ in Eq. (2) for losses due to imperfect dielectrics, this quantity is initially written as

$$\bar{P}_{d,d} = \sum_{i=1}^{N_D} \iint_{A_i} \omega\epsilon_i'' |E|^2\, dS, \tag{18}$$

where the second subscript d on $\bar{P}_{d,d}$ denotes that the dissipation losses are due to imperfect dielectrics, N_D is the number of imperfect dielectric regions in which the ith region has a complex permittivity given by

$$\hat{\epsilon}_i = \epsilon_i' - j\epsilon'', \tag{19}$$

A_i represents the transmission-line cross-sectional area spanned by the ith simply-connected homogeneous lossy dielectric region. Equation (18) can be rewritten as

$$\bar{P}_{d,d} = \sum_{i=1}^{N_D} \iint_{A_i} 2\omega(\tan\delta_i)\, \bar{W}_{ei}, \tag{20}$$

where

$$\tan\delta_i = \frac{\epsilon_i''}{\epsilon_i'} \tag{21}$$

and the time-averaged energy stored in the electric field in the ith dielectric region is given by

$$\bar{W}_{ei} = (1/2) \iint_{A_i} \epsilon_i' |E|^2\, dS. \tag{22}$$

In Appendix G details are presented of a development which shows that the time average of the total energy stored in the electric field per unit length of transmission line $\bar{W}_{e}$ is related to $\bar{W}_{ei}$ by

$$\bar{W}_{ei} = \epsilon_i \frac{\partial \bar{W}_e}{\partial \epsilon_i}\ . \tag{23}$$

The development in Appendix G provides a general result for transmission structures with many conductors and dielectrics over the cross section, but for isolated lines and coupled lines treated by an even- and odd-mode two-port interpretation $\bar{W}_e$ can be expressed as

$$\bar{W}_e = (1/2)C\,|V|^2, \tag{24}$$

where C is the electrostatic capacitance of the transmission line in the two-port configuration (isolated and even- or odd-mode line) and V is the voltage associated with a $+z$-traveling wave on the line.

To facilitate the evaluation of a dielectric-loss coefficient in terms of readily computable parameters, use is made of the definition of effective relative permittivity; hence

$$\epsilon_{\text{eff}} = \frac{C}{C_0}\,, \tag{25}$$

where C_0 is the electrostatic capacitance of the transmission line under consideration but with all dielectric materials fictitiously removed. Consequently $\bar{P}_{d,d}$ in Eq. (20) can be written as

$$\bar{P}_{d,d} = \omega C_0\,|V_0|^2 e^{-2\alpha z} \sum_{i=1}^{N_D} \epsilon_i(\tan\,\delta_i)\,\frac{\partial\epsilon_{\text{eff}}}{\partial\epsilon_i}\,. \tag{26}$$

By use of Eqs. (26), (2), and (6) the loss coefficient due to imperfect dielectrics can be written as

$$\alpha_d = \frac{\pi f}{c\sqrt{\epsilon_{\text{eff}}}} \sum_{i=1}^{N_D} \epsilon_i(\tan\,\delta_i)\,\frac{\partial\epsilon_{\text{eff}}}{\partial\epsilon_i}\ \text{(neper/unit length)} \tag{27}$$

or in units of dB per unit length,

$$\alpha_d = \frac{20\pi f}{c\,\ell n\,10\,\sqrt{\epsilon_{\text{eff}}}} \sum_{i=1}^{N_D} \epsilon_i(\tan\,\delta_i)\,\frac{\partial\epsilon_{\text{eff}}}{\partial\epsilon_i}\ \text{(dB/unit length).} \tag{28}$$

In Eqs. (27) and (28) c is the speed of light in free space.

To evaluate the partial derivative in Eqs. (27) and (28), a "forward" difference quotient is employed; thus

$$\frac{\partial\epsilon_{\text{eff}}}{\partial\epsilon_i} \approx \frac{\epsilon'_{\text{eff}} - \epsilon_{\text{eff}}}{\epsilon'_i - \epsilon_i}\,, \tag{29}$$

where ϵ'_{eff} is the value of ϵ_{eff} for the structure under consideration when the value of the relative permittivity for the ith homogeneous dielectric region is perturbed to a slightly different (higher) value ϵ'_i. Incorporating Eq. (29) into Eq. (28) provides the computationally useful result

$$\alpha_d \approx \frac{20\pi f}{c \, \ln 10 \sqrt{\epsilon_{\text{eff}}}} \sum_{i=1}^{N_D} \epsilon_i (\tan \delta_i) \left(\frac{\epsilon'_{\text{eff}} - \epsilon_{\text{eff}}}{\epsilon'_i - \epsilon_i} \right) \text{(dB/unit length)}. \tag{30}$$

It is this expression which has been incorporated into the computer programs which are documented in Appendixes A through E. For these programs the index i takes the value 1, since there is only one lossy homogeneous dielectric region in the overall inhomogeneous structures treated by these computer programs.

In the next subsection a description is given for the determination of conductor- and dielectric-loss effects in coupled line structures, with emphasis on directional couplers employing such configurations.

Coupled-Transmission-Line Structures

For the purposes of this subsection the coupled transmission lines will be treated by an even- and odd-mode interpretation [7]. Although two transmission lines coupled over a given length truly represent a four-port structure, the even and odd modes associated with this structure are each two-port transmission lines which can be treated separately. To properly assess the effects of losses in such structures, the problem is twofold. One problem is to determine the loss coefficients α_c and α_d for each of the even and odd modes. The second problem is to determine the effects of these coefficients on the four-port terminal electrical performance of the entire structure. In the material to follow the problem of determining the loss coefficients for each mode will be discussed first and will be followed by a treatment of the problem of determining their effects explicitly for four-port directional coupler losses.

Even- and Odd-Mode Loss Coefficients

Since the even and odd modes for a coupled line structure can each be depicted as two-port transmission lines whose length is that of the coupled-line region, the loss coefficients for conductor and dielectric losses can be determined using Eqs. (16) (or (17)) and (30) respectively. The even- and odd-mode loss coefficients for losses due to imperfect conductors can be written as

$$\alpha_{ck} \approx \frac{\dfrac{4\pi^2 (\epsilon_{\text{eff}})_k}{\mu_0} \sqrt{\dfrac{\pi f \mu_0}{\sigma}}}{2 \, |V_0|^2 \, v_k C_k} \sum_{j=1}^{N_c} \sum_{l=1}^{(N_S)_j} \left(q_{ik}^{N_j} \right)^2 \Delta \ell_{ik}^{N_j} \text{ (neper/unit length)}, \tag{31}$$

where $k = e$ for the odd mode and $k = 0$ for the odd mode. The quantities v_k, C_k, $q_{ik}^{N_j}$, $(\epsilon_{eff})_k$ and $\Delta \ell_{ik}^{N_j}$ are evaluated as described in Ref. 3. The expression for even- and odd-mode loss coefficients for losses due to imperfect dielectrics can be expressed as

$$\alpha_{dk} \approx \frac{20\pi f}{c \, \ell n \, 10 \sqrt{(\epsilon_{eff})_k}} \sum_{i=1}^{N_D} \epsilon_i (\tan \delta_i) \left[\frac{(\epsilon'_{eff})_k - (\epsilon_{eff})_k}{\epsilon'_i - \epsilon_i} \right] \text{(dB/unit length)}, \quad (32)$$

where $k = e$ for the even mode and $k = 0$ for the odd mode. In Eq. (32) $(\epsilon_{eff})_k$ represents the effective relative permittivity for the even or odd mode in the coupled-line configuration to be analyzed. This value is determined using the method described explicitly in Ref. 3. The value $(\epsilon'_{eff})_k$ is determined the same way for the coupled-line structure in which the relative permittivity of the ith dielectric region ϵ_i is perturbed to the slightly higher value ϵ'_i. Equations (31) and (32) have been used to evaluate the even- and odd-mode conductor- and dielectric-loss coefficients for the coupled-line structures described later in this report. The total-loss coefficients α_e and α_o, for the even and odd modes, are obtained by adding the respective values of α_{ck} and α_{dk}.

Effects of Losses on Four-Port Terminal Performance

The effects of losses on four-port terminal performance of directional couplers can be understood by considering Fig. 1. This figure portrays the problem as being that of determining the loss effects on measurable terminal voltages b_i (i = 1, 2, 3, 4) once the loss coefficients (α_{ce}, α_{de}) and (α_{co}, α_{do}) have been determined for the modal transmission lines in Figs. 1b and 1c respectively. These loss coefficients are assumed to be known for this development, having been computed by the method described in the previous section.

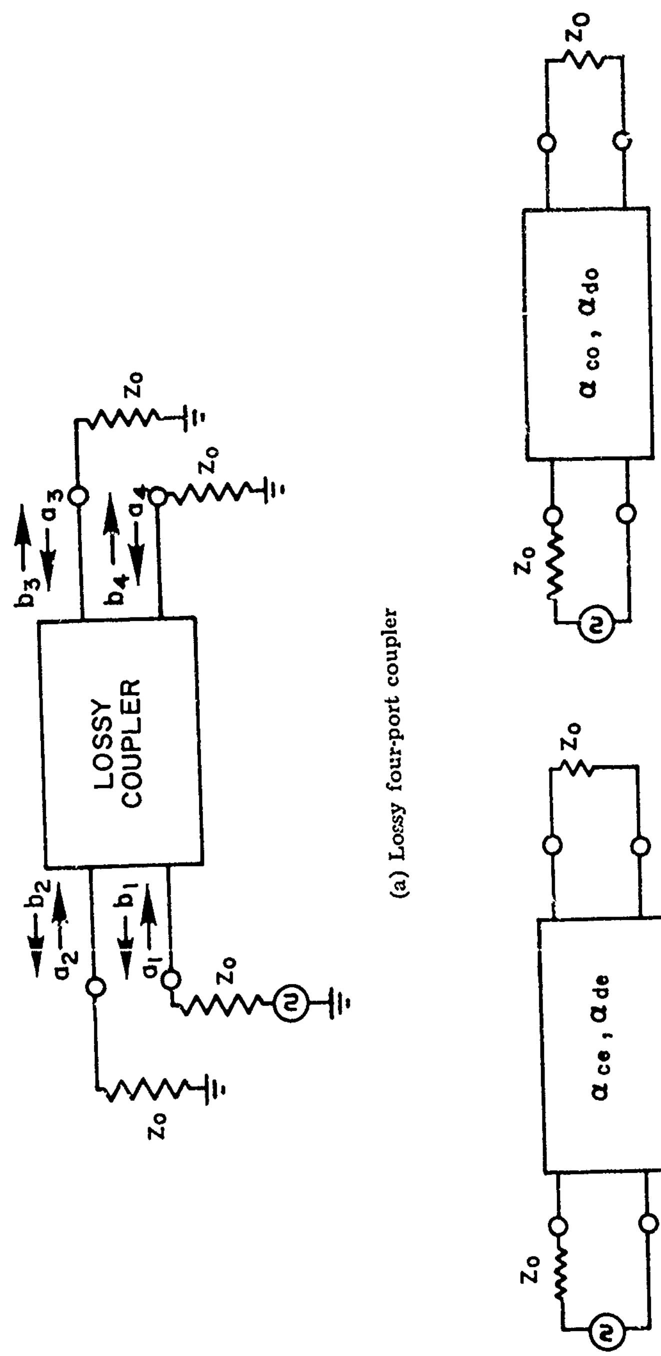
The approach used here starts with a procedure similar to that described in Ref. 7. The voltages b_i (i = 1, 2, 3, 4) are written as

$$b_1 = \frac{\Gamma_{oe} + \Gamma_{oo}}{2} = \text{reflected signal} \equiv b_r, \quad (33)$$

$$b_2 = \frac{\Gamma_{oe} - \Gamma_{oo}}{2} = \text{coupled signal} \equiv b_c, \quad (34)$$

$$b_3 = \frac{T_{oe} - T_{oo}}{2} = \text{isolated signal} \equiv b_i, \quad (35)$$

$$b_4 = \frac{T_{oe} + T_{oo}}{2} = \text{transmitted signal} \equiv b_t, \quad (36)$$



(a) Lossy four-port coupler

(b) Even-mode two-port network

(c) Odd-mode two-port network

Fig. 1 — Schematic representation of the treatment of four-port coupler loss

where (Γ_{oe}, T_{oe}) and (Γ_{oo}, T_{oo}) are pairs of reflection and transmission coefficients for the even- and odd-mode two-port transmission lines respectively. The reflection coefficients for the even or odd mode can be expressed in terms of two-port-network parameters A, B, C, and D as

$$\Gamma_{oi} = \frac{A_i + \dfrac{B_i}{Z_0} - C_i Z_0 - D_i}{A_i + \dfrac{B_i}{Z_0} + C_i Z_0 + D_i}, \tag{37}$$

where $i = e$ for the even mode and $i = 0$ for the odd mode. Similarly, the transmission coefficients can be expressed as

$$T_{oe} = \frac{2}{A_i + \dfrac{B_i}{Z_0} + C_i Z_0 + D_i}, \tag{38}$$

where again $i = e$ for the even mode and $i = 0$ for the odd mode. In Eqs. (37) and (38) Z_0 represents the characteristic impedance of the input and output ports shown in Fig. 1 ($Z_0 \approx \sqrt{Z_{oe} Z_{oo}}$). The parameters A, B, C, and D for the lossy even-mode two-port are given by

$$A_d = D_e = \cosh \gamma_e \ell, \tag{39}$$

$$B_e = Z_{oe} \sinh \gamma_e \ell, \tag{40}$$

$$C_e = \left(\frac{1}{Z_{oe}}\right) \sinh \gamma_e \ell. \tag{41}$$

Similarly, for the odd mode

$$A_o = D_o = \cosh \gamma_o \ell, \tag{42}$$

$$B_o = Z_{oo} \sinh \gamma_o \ell, \tag{43}$$

$$C_o = \left(\frac{1}{Z_{oo}}\right) \sinh \gamma_o \ell. \tag{44}$$

In Eqs. (39) through (44) ℓ represents the physical length of the coupled-line region, Z_{oe} and Z_{oo} are the characteristic impedances of the even- and odd-mode transmission lines respectively, and γ_e and γ_o are the even- and odd-mode propagation constants, taken for this development to be given by

$$\gamma_e = \alpha_e + j\beta_e, \tag{45}$$

$$\gamma_o = \alpha_o + j\beta_o, \tag{46}$$

where α_e and α_o are the total (known) even- and odd-mode loss coefficients and β_e and β_o are the even- and odd-mode phase constants (known) respectively, which are determined by the lossless analysis described in detail in Ref. 3. The generality of this development permits β_e and β_o to be determined with allowance for even and odd modes having different phase velocities.

By properly combining Eqs. (33) through (44), the measurable terminal voltages of the four-port coupler under analyses can be expressed as

$$b_c = F_2 \left(\frac{\tanh \gamma_e \ell}{2 + F_1 \tanh \gamma_e \ell} + \frac{\tanh \gamma_o \ell}{2 + F_1 \tanh \gamma_o \ell} \right), \tag{47}$$

$$b_t = \frac{\operatorname{sech} \gamma_e \ell}{2 + F_1 \tanh \gamma_e \ell} + \frac{\operatorname{sech} \gamma_o \ell}{2 + F_1 \tanh \gamma_o \ell}, \tag{48}$$

$$b_i = \frac{\operatorname{sech} \gamma_e \ell}{2 + F_1 \tanh \gamma_e \ell} - \frac{\operatorname{sech} \gamma_o \ell}{2 + F_1 \tanh \gamma_o \ell}, \tag{49}$$

$$b_r = F_2 \left(\frac{\tanh \gamma_e \ell}{2 + F_1 \tanh \gamma_e \ell} - \frac{\tanh \gamma_o \ell}{2 + F_1 \tanh \gamma_o \ell} \right). \tag{50}$$

In Eqs. (47) through (50) F_1 and F_2 are quantities given by

$$F_1 \equiv \left(\frac{Z_{oe}}{Z_{oo}} \right)^{1/2} + \left(\frac{Z_{oo}}{Z_{oe}} \right)^{1/2}, \tag{51}$$

$$F_2 \equiv 1/2 \left[\left(\frac{Z_{oe}}{Z_{oo}} \right)^{1/2} - \left(\frac{Z_{oo}}{Z_{oe}} \right)^{1/2} \right]. \tag{52}$$

For the special case in which $\beta_e \ell = \beta_o \ell = \pi/2$, Eqs. (47) through (50) simplify

$$b_c = F_2 \left(\frac{1}{2\alpha_e \ell + F_1} + \frac{1}{2\alpha_o \ell + F_1} \right), \tag{53}$$

$$b_t = -j \left(\frac{1}{2\alpha_e \ell + F_1} + \frac{1}{2\alpha_o \ell + F_1} \right), \tag{54}$$

$$b_i = -j\left(\frac{1}{2\alpha_e\ell + F_1} - \frac{1}{2\alpha_o\ell + F_1}\right), \tag{55}$$

$$b_r = F_2\left(\frac{1}{2\alpha_e\ell + F_1} - \frac{1}{2\alpha_o\ell + F_1}\right). \tag{56}$$

In Eqs. (53) through (56) the even- and odd-mode loss coefficients appear explicitly. Also, these relationships reveal that, even for perfectly matched couplers, for α_e different from α_o the isolated and reflected signals are not zero. For couplers with other sources of impedance match and isolation degradation (such as different even- and odd-mode phase velocities) the additional degradation of isolation and reflected signal due to different modal loss characteristics will be superimposed on the other effects. The relationships developed in this section have been tested on experimental coupler models, and the results are described in the next section.

EXAMPLES OF LOSS EVALUATIONS AND DESIGN INFORMATION

In this section results are presented for specific structures of isolated and coupled transmission lines. The examples of microstrip and coplanar waveguide are presented to demonstrate the accuracy of the computer-aided analysis from the preceding section and to provide useful design information for engineering applications. The results for inverted microstrip and trapped inverted microstrip serve to provide useful, currently unavailable design information for more complicated structures which offer potential for circuit applications. Following these results is a comparison of the loss characteristics for the four preceding transmission lines. The results for the edge-coupled microstrip structure with a dielectric overlay confirm the utility of the coupled-line loss analysis described in the preceding section and provide currently unavailable design information for this structure. This structure is presently the most viable approach for providing broadband couplers, filters, and Schiffman-phase-shift sections in a microstrip-compatible format.

Isolated Transmission Lines

Microstrip

To illustrate the approach that resulted in Eq. (16), the conductor-loss coefficient was computed for a microstrip line with an impedance of 50 ohms (nominally). A generic cross section of such a line is portrayed in Fig. 2. The dc conductivity, for Eq. (16), was selected to be 4.10×10^7 mho/m based on data provided in Ref. 8, corresponding to the value designated for gold. The values of α_c computed for this configuration are represented by the solid curve in Fig. 3. The reference values shown, represented by a broken curve, are due to a computer-program version of Schneider's results [2]. Schneider's results were selected as a reference because they have compared favorably with the experimental data provided in Refs. 9 and 10. The agreement here is reasonable, with the values computed by the method proposed in this report being only a few hundredths dB per

wavelength higher than the reference. This discrepancy can be attributed to the discrete nature of the equivalent charge densities used in the formulation. This effect is somewhat equivalent to a surface-roughness effect.

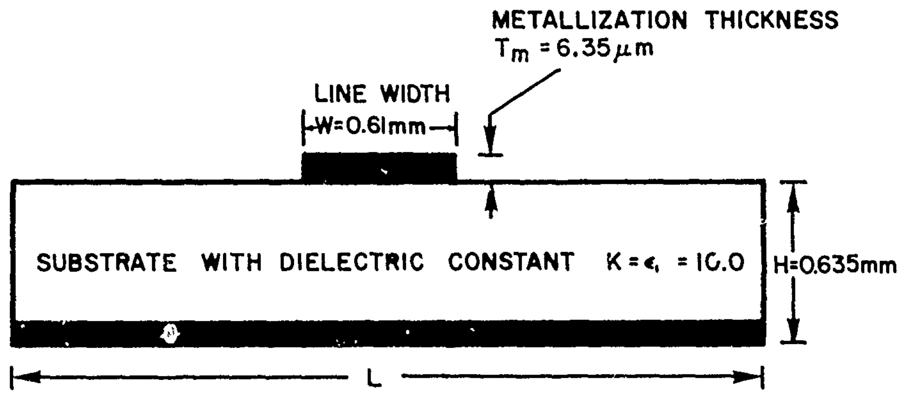


Fig. 2 — Generic cross section of nominal-50-ohm microstrip line

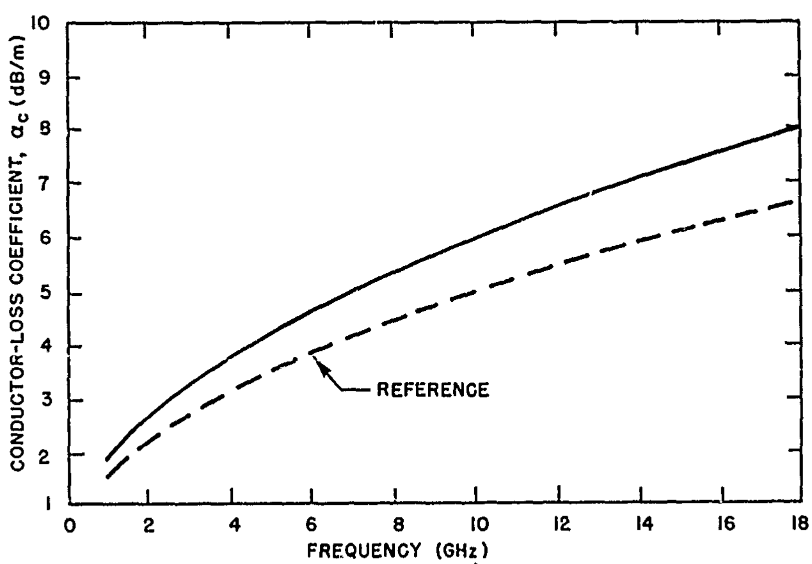


Fig. 3 — Conductor attenuation coefficient for microstrip as configured in Fig. 2 (a configuration with Z_0 = 50 ohms).

The method for evaluating dielectric-loss effects was tested for the same microstrip configuration. Equation (30) was employed with N_D equal to 1 and the loss tangent value, tan δ_1, selected to be 6×10^{-4} (alumina). The results computed using Eq. (30) are represented in Fig. 4 by the solid curve. Shown for comparison are reference values due to

Pucel et al. [9], which are in excellent agreement. The method described for computing dielectric losses that resulted in Eq. (30) is similar to that of Schneider [11]. In Ref. 12 Simpson and Tseng show additional results which agree within 1% (typically) compared to results computed by this method.

The total-loss coefficient (sum of α_c and α_d) for the microstrip line under consideration will later be compared (in Fig. 15) with similar characteristics for nominally-50-ohm configurations of coplanar waveguide, inverted microstrip, and trapped inverted microstrip.

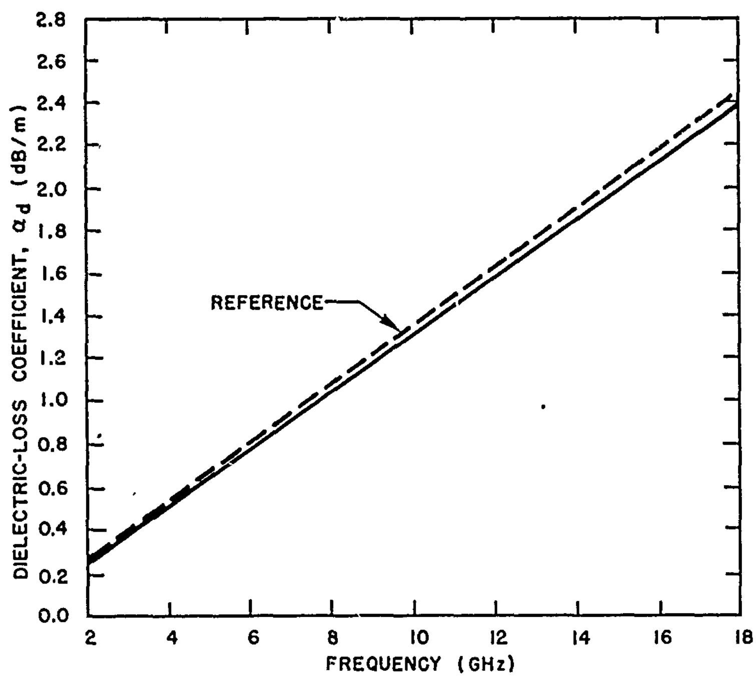


Fig. 4 — Dielectric-loss coefficient for microstrip as configured in Fig. 2 (a configuration with Z_0 = 50 ohms).

Coplanar Waveguide

The second illustrative example chosen to demonstrate the effectiveness of the methods that were described in the analysis section is coplanar waveguide [1] with the cross section depicted in Fig. 5. The computations here were for an approximately-50-ohm configuration. The metallization thickness was again selected to be 6.35 μm with a dc conductivity of 4.10 $\times$ 10^7 mho/m. Figure 6 shows the computed α_c for this case plotted versus frequency as a solid curve. The computed α_d for this configuration is also shown in Fig. 6, plotted as a broken curve. The total-loss coefficient, obtained by summing α_c and α_d, is shown plotted in Fig. 7 (and Fig. 15). Also shown in Fig. 7 are measured values [13] for comparison. These values were obtained from end-coupled resonant-section transmission measurements. The agreement in this frequency range was reasonable, being

within 0.1 dB per wavelength over most of the range from 4 to 11 GHz. Equally good agreement (about 0.1 dB/wavelength or better) was found between the computed values of total loss and values obtained by a straight-through transmission-line measurement [13]. This correlation in values was obtained from 4 GHz through about 10 GHz.

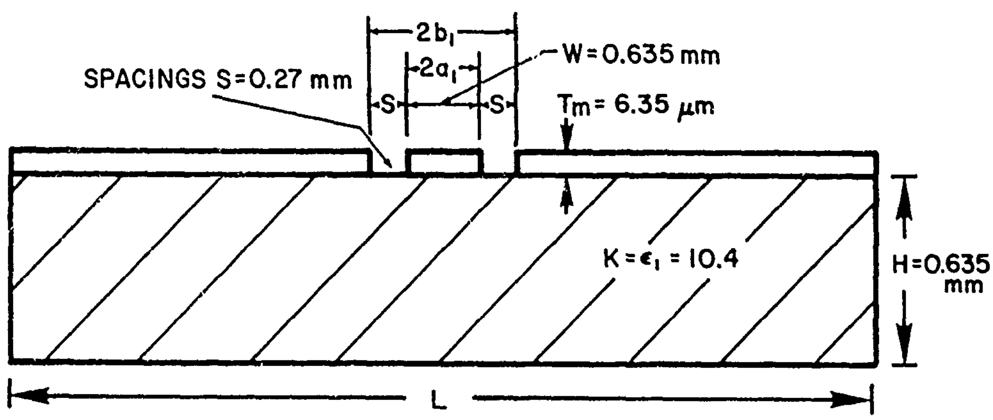


Fig. 5 — Generic cross section of nominal-50-ohm coplanar waveguide

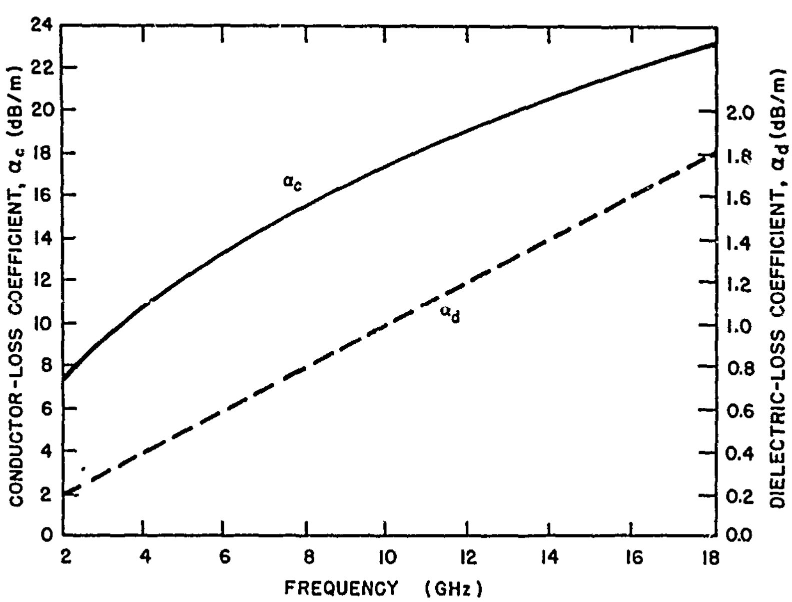


Fig. 6 — Conductor- and dielectric-loss coefficients for coplanar waveguide configured (as in Fig. 5) with Z_0 = 50 ohms

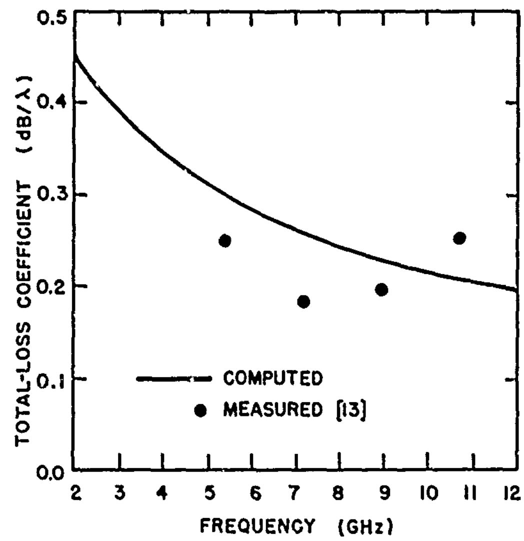


Fig. 7 — Total loss coefficient versus frequency for coplanar waveguide

Additional design information developed for coplanar waveguide, using the methods described in the analysis section, are shown in Table 1. The information in this table relating to configuration geometry and material characteristics is consistent with the notation defined by Fig. 5. The ratios of a_1/b_1 shown were selected to span a range of values that should impact on a broad range of applications for this transmission line.

Inverted Microstrip

Inverted microstrip is the third transmission line which has been analyzed using the method described in the analysis section. The generic cross section of inverted microstrip is depicted in Fig. 8. Table 2 shows the analysis results for aspect ratios (W/H) spanning a range of values from 1 to 8. This range was chosen to provide design information for a broad range of circuit applications. The substrate dielectric constant chosen for the set of data corresponds to that of fused silica, which is deemed to be a suitable material for this transmission line for applications at higher microwave and millimeter-wave frequencies. The conductor- and dielectric-loss coefficients evaluated for this sequence of transmission-line configurations are plotted as a function of aspect ratio W/H in Fig. 9 for design usage. The characteristic impedances and phase velocities for configurations spanning the range of aspect ratios W/H from 1 to 8 are plotted in Fig. 10, readily usable for circuit design applications.

To demonstrate the correlation between computed and measured characteristics, the computed phase velocity is compared to experimental data for inverted microstrip. Figure 11 shows computed values of $\sqrt{\epsilon_{eff}}$ versus aspect ratio W/H as a solid curve. Measured points shown in this figure were obtained via time-domain reflectometer measurements [2]. The errors between measured and computed values are within 3% over this range of aspect

ratios. (The total-dissipation-loss coefficient for a nominally-50-ohm inverted-microstrip configuration is plotted in Fig. 15 for comparison with the other transmission lines.)

Table 1 — Design Information for Coplanar Waveguide as Shown in Fig. 5 But for a Range of W and S Values Rather Than the Nominal-50-Ohm Values (and With H = 0.635 mm, ϵ_1 = 10.0, ϵ' = 10.1, and tan δ_1 = 0.0006)

a_1/b_1	W (mm)	S (mm)	Z_0 (ohms)*	v (10^8 m/s)*	$\alpha_c/\sqrt{f}$ (10^{-4} dB/m$\sqrt{\text{Hz}}$)	α_d/f (10^{-11} dB/m Hz)
0.1	0.118	0.529	102	1.36	1.32	10.0
0.3	0.353	0.412	72.2	1.37	1.22	11.82
0.5	0.588	0.294	57.5	1.38	1.51	9.41
0.7	0.823	0.176	46.2	1.38	2.29	7.56

*Tabulated electrical characteristic determined by the method described in Ref. 3. This characteristic is furnished in the output of computer program furnished in Appendix B.

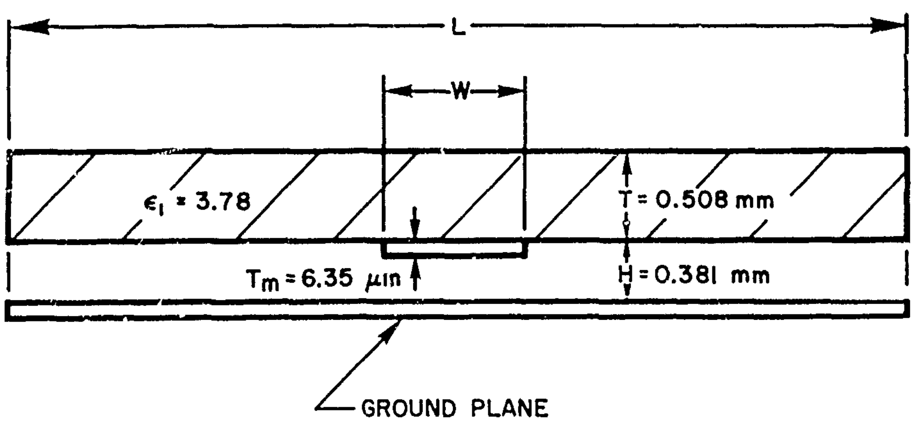


Fig. 8 — Generic cross section of nominal-50-ohm inverted microstrip

Table 2 — Design Information for Inverted Microstrip as Shown in Fig. 8 But for a Range of W/H Values (with H, T, T_m, and ϵ_1 as Shown and $\epsilon_1' = 3.88$, $\tan \delta_1 = 0.0002$, and a Metal dc Conductivity of 4.10×10^7 mho/m.)

W/H	W (mm)	Z_0 (ohms)*	v (10^8 m/s)*	$\alpha_c/\sqrt{f}$ (10^{-5} dB/m$\sqrt{\text{Hz}}$)	α_d/f (10^{-12} dB/m Hz)
1	0.381	101.8	2.41	2.74	8.40
2	0.762	75.1	2.52	2.37	6.03
3	1.143	60.8	2.60	2.26	5.02
4	1.524	51.4	2.66	2.19	4.24
5	1.905	44.7	2.70	2.16	3.69
6	2.286	39.6	2.73	2.14	3.27
7	2.667	35.6	2.76	2.13	2.94
8	3.048	32.4	2.78	2.11	2.67

*Tabulated electrical characteristic determined by the method described in Ref. 3. This characteristic is furnished in the output of the computer program furnished in Appendix C.

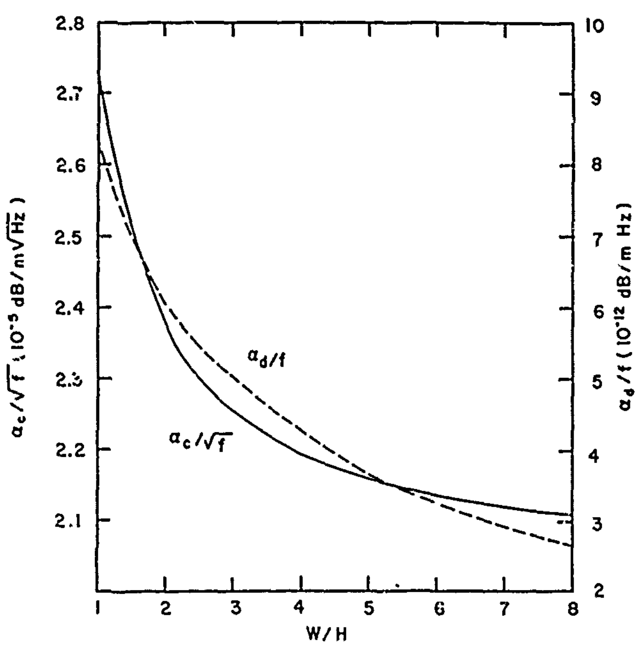


Fig. 9 — Loss constants vs aspect ratio for trapped inverted microstrip (Fig. 8)

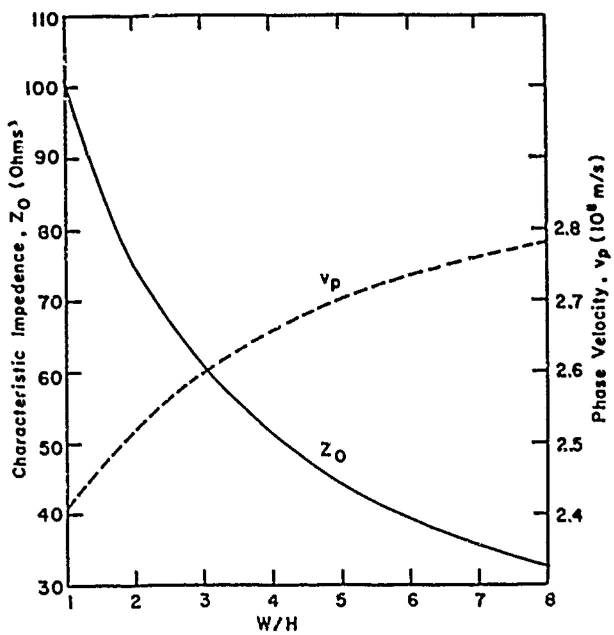


Fig. 10 — Characteristic impedance Z_0 and phase velocity v_p vs W/H for trapped inverted microstrip (Fig. 9)

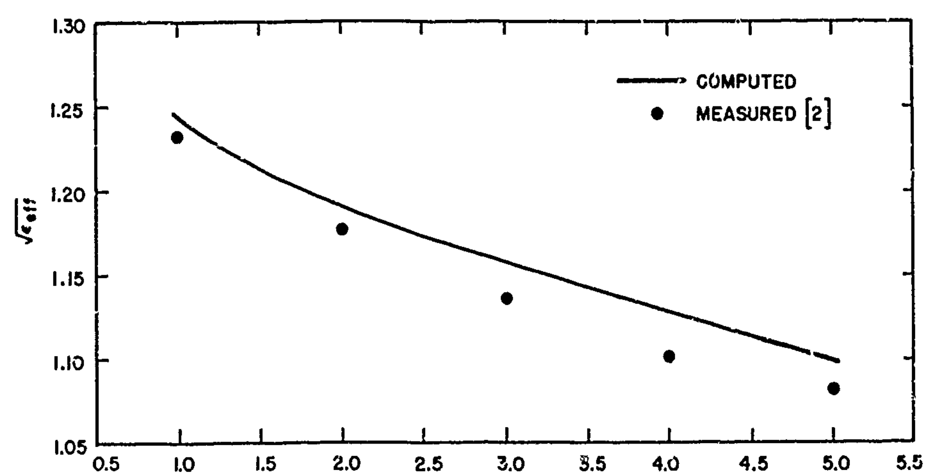


Fig. 11 — Computed and measured $\sqrt{\epsilon_{eff}}$ versus W/H characteristic for inverted microstrip (Fig. 8)

Trapped Inverted Microstrip

This section provides design information for trapped inverted microstrip, characterized by the generic cross-section configuration shown in Fig. 12. Table 3 provides details of configuration geometry and material parameters for the cross sections analyzed in this work. So that this information can be used for design purposes, the conductor- and dielectric-loss coefficients are plotted in Fig. 13 for aspect ratios W/H ranging from 0.25 to 6.0. The somewhat irregular appearance of these characteristics can be attributed to the effects of channel-sidewall interaction with the conducting strip, which is not explicitly accounted for by the "aspect ratio" definition as W/H. A similar situation is apparent in Fig. 14, where the characteristic impedance Z_0 and phase velocity v_p are shown for the same range of W/H values.

The total dissipation-loss coefficient for a nominally-50-ohm configuration of trapped inverted microstrip is plotted in Fig. 15 for comparison with those of microstrip, coplanar waveguide, and inverted microstrip.

Comparison of Loss Characteristics

This section presents a comparison of the loss characteristics for specific nominally-50-ohm configurations of microstrip, coplanar waveguide, inverted microstrip, and trapped inverted microstrip transmission lines. The total-loss coefficients (sum of the conductor- and dielectric-loss coefficients) for these lines are shown in Fig. 15 for the frequency range from 0 to 60 GHz. Effects and consideration of higher order moding have been neglected, as well as effects due to radiation losses. Details defining the configurations of microstrip and coplanar waveguide analyzed to provide the corresponding curves in this figure are shown in Figs. 2 and 5 respectively, with the dc conductivity σ selected to be 4.10×10^7 mho/m for both configurations. The configuration of the inverted microstrip line analyzed is defined by Fig. 8, with W = 1.676 mm, ϵ_1' = 3.88, $\sigma = 4.10 \times 10^7$ mho/m, and $\tan \delta_1 = 2 \times 10^{-4}$. The configuration for the trapped inverted microstrip line analyzed is specified by the following values in conjunction with the nomenclature shown in Fig. 12: W = 1.52 mm, S = 1.27 mm, H = 0.508 mm, T_m = 6.35 μm, ϵ_1 = 10.0, ϵ_1' = 10.1, $\sigma = 4.10 \times 10^7$ mho/m, and $\tan \delta_1 = 6 \times 10^{-4}$.

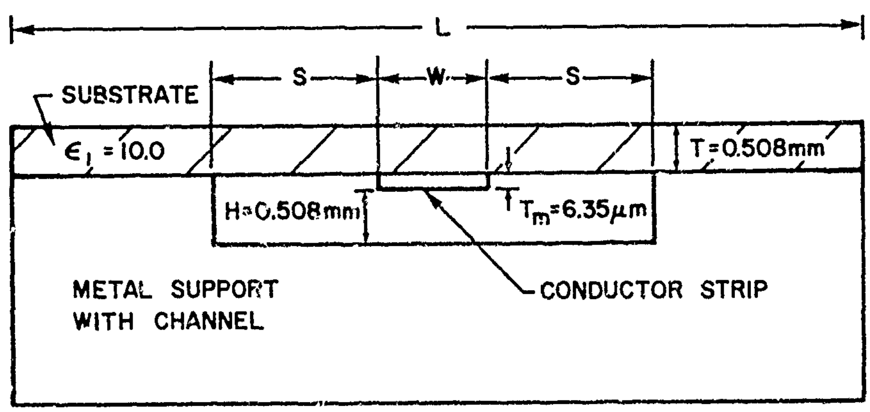


Fig. 12 — Generic cross section for nominal-50-ohm trapped inverted microstrip

Table 3 — Design Information for Trapped Inverted Microstrip as Shown in Fig. 12 But for a Range of W/H Values (With H, T, T_m, and ϵ_1 as Shown and With ϵ_1' = 10.1, tan δ_1 = 0.0006, and a Metal dc Conductivity of 4.10 × 10^7 mho/m)

W/H	W (mm)	S (mm)	Z_0 (ohms)*	v_p (10^{-8} m/s)*	$\alpha_c/\sqrt{f}$ (10^{-5} dB/m$\sqrt{\text{Hz}}$)	α_d/f (10^{-11} dB/m Hz)
0.25	0.127	1.97	122	1.77	3.61	5.81
0.5	0.254	1.90	102	1.83	2.72	5.32
1	0.508	1.78	80.7	1.92	2.25	4.76
2	1.02	1.52	61.6	1.97	2.22	4.44
3	1.52	1.27	49.0	2.12	1.97	3.85
4	2.03	1.02	41.6	2.18	2.00	4.08
5	2.54	0.762	35.0	2.18	2.19	4.02
6	3.05	0.508	29.5	2.13	2.55	4.35

*Tabulated electrical characteristic determined by the method described in Ref. 3. This characteristic is furnished in the output of the computer program furnished in Appendix D.

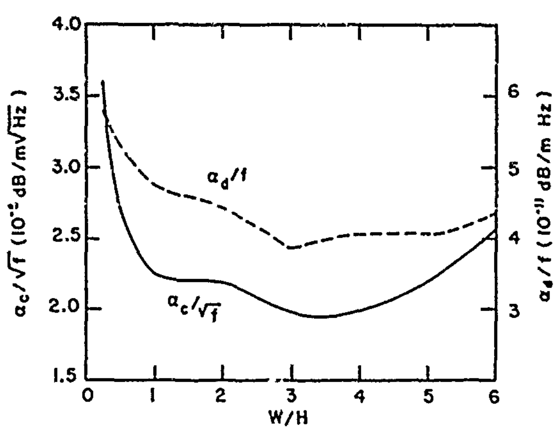


Fig. 13 — Loss constants vs aspect ratio for trapped inverted microstrip (Fig. 12)

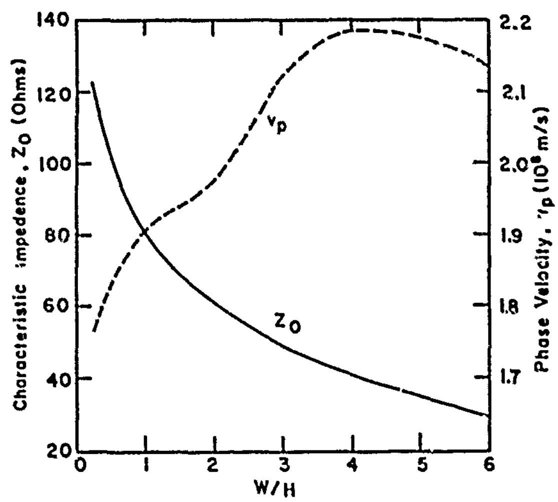


Fig. 14 — Characteristic impedance Z_0 and phase velocity v_p vs W/H for trapped inverted microstrip

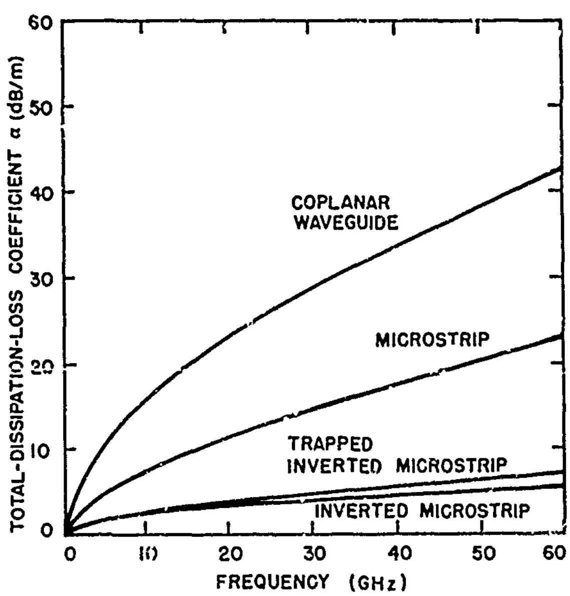


Fig. 15 — Comparison of total dissipative losses for four 50-ohm transmission lines

The microstrip configuration chosen here represents a standard configuration of this transmission line as it has been used in many applications at frequencies up through about 12 GHz. This line is representative of the transmission line fabricated using an alumina substrate with a predominantly gold metallization system. The same is true for the selected

coplanar waveguide configuration. The configuration for inverted microstrip was chosen to represent a version fabricated on a fused-silica substrate, again with gold being the predominant carrier of RF current in the metallization system. The configuration for trapped inverted microstrip represents a model fabricated on a standard alumina substrate, metallized with a predominantly gold system.

Coupled Transmission Lines

Edge-Coupled Microstrip with a Dielectric Overlay

This section serves a twofold purpose. The first purpose is to provide an illustrative example demonstrating the applicability of the coupled-line loss analysis developed in the analysis sections. The second purpose is to provide a quantitative assessment of the losses encountered in quarter-wavelength-long (at midband) sections of edge-coupled microstrip line with a dielectric overlay for various coupling levels. This coupled-line structure is currently the most viable approach for providing high-performance broadband couplers, filters, and Schiffman-phase-shift sections in a microstrip-compatible format [14].

The generic cross section of the edge-coupled microstrip with dielectric overlay is portrayed in Fig. 16. Four configurations differing in geometric parameters were analyzed (Table 4). Table 5 lists the computed electrical parameters for these sections.

Sections 1 and 2 in these tables were fabricated into a two-section, asymmetric coupler similar to that shown in Fig. 17. The circuit pattern was etched on a portion of alumina substrate with dimensions 23 mm by 10 mm by 0.635 mm. The metallization was chromium-gold with a thickness of 6.35 μm. The overlays were made of alumina pieces on each section and were attached to the substrate using Stycast Hi K epoxy (K = 10). The two-section coupler, over the frequency band from 2 to 8.5 GHz, has a nominal coupling value of 6.7 dB. Using Eqs. (53) through (56) and the computed values of loss coefficient shown in Table 5 the calculated dissipation loss for the two coupled-line sections was found to be 0.2 dB, obtained for the coupler midband frequency of 5.4 GHz. When measured losses due to connectors and theoretical lead-in-line losses are added, the total dissipation loss determined is 0.42 dB. This agrees well with the total measured coupler dissipation of 0.4 dB.

Sections 3 and 4 in Tables 4 and 5 were combined in the fabrication of a two-section asymmetric coupler similar to the coupler shown in Fig. 17. The experimental model had a nominal coupling characteristic of 20 dB over the frequency band from 2 to 8.5 GHz. Using the evaluation scheme described for the coupler in Fig. 17, the dissipation loss for the coupled line sections was determined to be 0.08 dB. After adding contributions for connector and lead-in line lengths, the total dissipation loss was computed to be 0.38 dB. This agrees well with the measured midband (5.4 GHz) dissipation loss of 0.4 dB.

Another portrayal of the loss characteristics computed for the four coupled-line sections of Tables 4 and 5 is shown in Figs. 18 and 19. Figure 18 shows plots of α_{co} and α_{do} versus frequency for the coupling sections used in the 6.7-dB coupler. Also shown are the ratios α_{co}/α_{ce} and α_{do}/α_{de}. A large disparity exists between the even- and odd-mode conductor losses for the tight section. Figure 19 shows similar data for the coupling sections used to make up the 20-dB coupler.

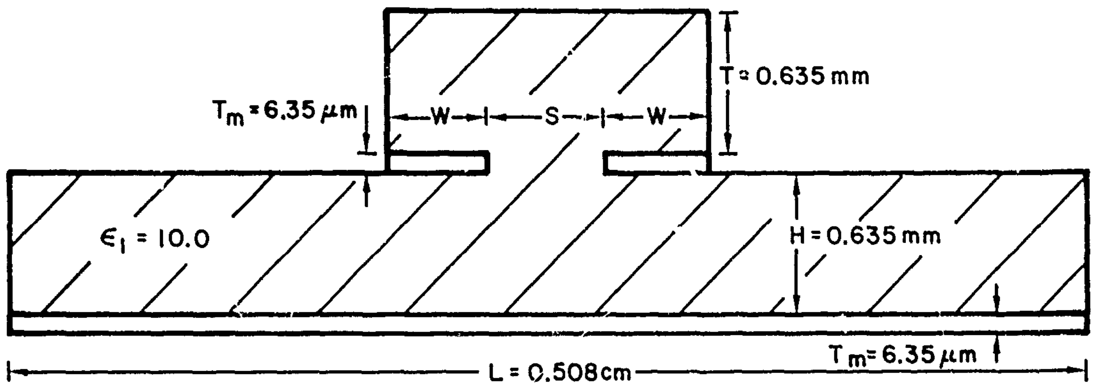


Fig. 16 — Generic cross section of nominal-50-ohm edge-coupled microstrip with a dielectric overlay

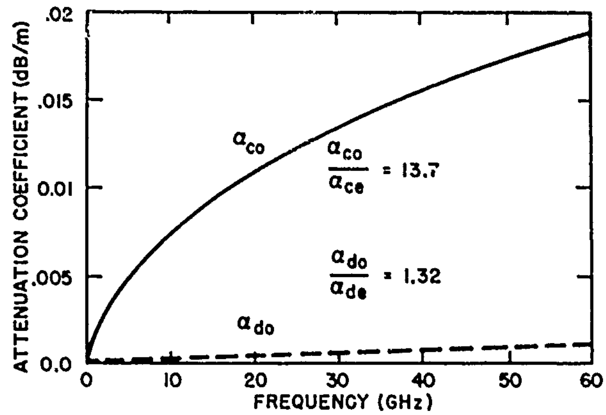
Table 4 — Parameters Which Along With H, L, T_m, and ϵ_1 as Shown in Fig. 16 and With $\epsilon_1' = 10.1$, $\tan \delta_1 = 0.0006$, and $\sigma = 4.10 \times 10^7$ mho/m Define Four Configurations of Edge-Coupled Microstrip With Dielectric Overlay

Section	W (mm)	S (mm)
1	0.223	0.0305
2	0.483	0.541
3	0.483	0.927
4	0.467	2.31

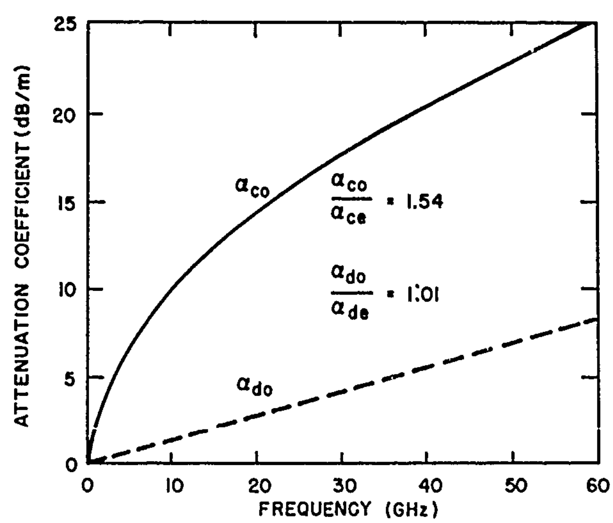
Table 5 — Computed Electrical Parameters for the Coupled-Line Sections Described by Table 4

Section	Lossless Midband Coupling (dB)	Z_0 (ohms)	Z_{oe} (ohms)	Z_{oo} (ohms)	$\alpha_{ce}/\sqrt{f}$ (10^{-5} dB/m$\sqrt{\text{Hz}}$)	$\alpha_{co}/\sqrt{f}$ (10^{-5} dB/m$\sqrt{\text{Hz}}$)	α_{dc}/f (10^{-10} dB/m Hz)	α_{do}/f (10^{-10} dB/m Hz)
1	3.3	50.0	115	21.7	5.60	76.9	1.20	1.59
2	12.8	50.7	64.1	40.2	6.69	10.3	1.36	1.38
3	16.9	50.7	58.6	43.9	7.59	9.31	1.60	1.38
4	26.8	49.4	51.7	47.2	9.77	10.2	1.90	1.13

Fig. 17 — Three partial assemblies of a two-section coupler employing edge-coupled microstrip with a dielectric overlay.

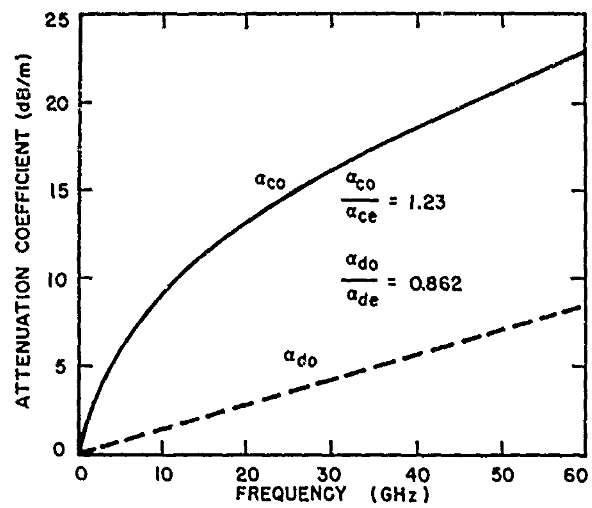


(a) Tight section (section 1)

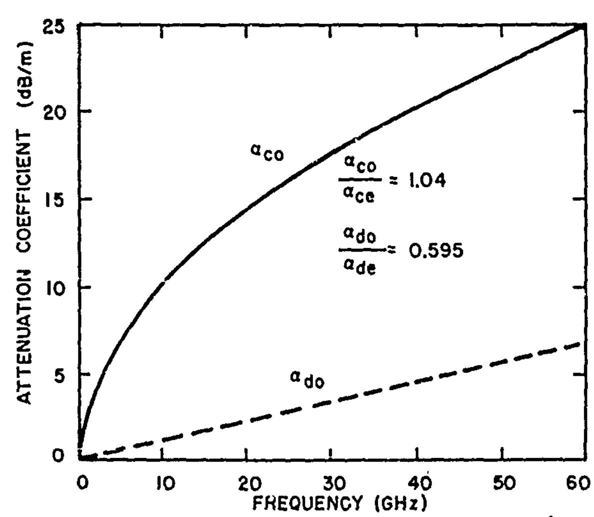


(b) Loose section (section 2)

Fig. 18 — Loss coefficients for the sections of the 6.7-dB two-section coupler that combines sections 1 and 2 of Tables 4 and 5



(a) Tight section (section 3)



(b) Loose section (section 4)

Fig. 19 — Loss coefficients for the sections of the 20-dB two-section coupler that combines sections 3 and 4 of Tables 4 and 5

DISCUSSION

This report details and illustrates analyses, amenable to computer programming, which can evaluate dissipation losses in isolated or coupled microwave and millimeter-wave transmission lines. These lines can be composed of both imperfect conductors and piecewise homogeneous dielectric materials. The analyses described here have been implemented in the form of digital computer programs for treating microstrip, coplanar waveguide, inverted microstrip, trapped inverted microstrip, and edge-coupled microstrip with a dielectric overlay.

The programs for microstrip and coplanar waveguide were used to analyze various configurations for these transmission lines. The computed values of conductor loss for microstrip on alumina were compared with well-accepted values due to Schneider [2] and were found to agree within better than 1.6 dB/m for frequencies up through 18 GHz. The computed values for dielectric losses in microstrip agree with Simpson and Tseng [12] and Schneider [11] and agreed to within 3% or better at frequencies up to 18 GHz. The sum of conductor and dielectric losses computed for coplanar waveguide agreed with the experimental values due to McDade and Stockman [13] to within 0.1 dB/wavelength. The experimental determinations were made by two independent techniques.

Conductor- and dielectric-loss coefficients were also evaluated for inverted microstrip and trapped inverted microstrip. This information was presented in the form of useful design curves, providing loss values as functions of the aspect ratios for these lines. There is encouraging agreement (better than 2%) between computed phase velocities ($\sqrt{\epsilon_{eff}}$) for inverted microstrip and measured data [2]. These calculations are made using the same equivalent charge densities that are used in the loss calculations.

The computed total loss coefficients for the four types of lines are compared in Fig. 15. The losses incurred in inverted and trapped inverted microstrip at frequencies as high as 60 GHz are comparable to those incurred in microstrip at frequencies in the range 5 to 10 GHz. Also, coplanar waveguide appears to be considerably more lossy than microstrip. The structures compared in this figure were selected to have characteristic impedance levels of nominally 50 ohms. Consistent with this characteristic the conducting strip widths required for microstrip and coplanar waveguide were approximately 2-1/2 to 3 times narrower than those required for inverted and trapped inverted microstrip. This feature enhances the attractiveness of the inverted and trapped inverted microstrip lines by virtue of the mitigation of fabrication difficulties. By concentrating more of the field energies in air (with correspondingly lower ϵ_{eff}), wider strips are possible for a prescribed impedance level (compared to the other two lines). These advantages should be realized even if the dimensions must be contracted to suppress higher order moding at the higher frequencies. This should be investigated more extensively.

The analyses for computing conductor- and dielectric loss coefficients and for relating these parameters to terminal electrical characteristics were applied to edge-coupled microstrip with a dielectric overlay. Computed results were compared to measured characteristics for two experimental models of two-section couplers, having nominal coupling values of 6.7 dB and 20 dB respectively. Total dissipation values computed for these couplers agreed with experimental evaluations within 0.07 dB.

The difference between the even- and odd-mode loss coefficients for this structure gives rise to a finite isolation for an otherwise perfect coupled line section. For the 6.7-dB coupler considered here the tight and loose sections are limited to maximum isolation levels of 42 dB and 59 dB respectively due to this phenomenon. The tight and loose sections of the 20-dB coupler are similarly limited to 70-dB and 92-dB isolation levels respectively.

Several factors can contribute to error in the analyses described here. One is the discrete representation of charge-density distributions employed in the quasi-TEM model employed. In one sense this discretization can be viewed as a "surface roughness" which could lead to high estimates of losses. Mathematically smoothing the charge-density distributions might lead to improvement. Another source of error is the approximation shown in Eq. (9). A method which would reduce this error is to solve the magnetostatic problem of evaluating the magnetic field at conductor surfaces for these transmission media. Certainly the difference-quotient approximation, shown in Eq. (29), introduces error in the dielectric-loss evaluations. Also, judicious choices must be made for the dc conductivity of the metal system and the loss tangents for dielectrics.

In the next section documentation is given for the user-oriented computer programs which can provide design information for the structures treated in earlier sections.

COMPUTER PROGRAMS

This section describes the computer programs MS, CPW, IM, TIM, and LEMDOC, listed in Appendixes A, B, C, D, and E. These programs compute the electrical characteristics, including loss characteristics, for microstrip, coplanar waveguide, inverted microstrip, trapped inverted microstrip, and edge-coupled microstrip with a dielectric overlay respectively.

Program MS (Microstrip)

Program MS, which computes the characteristics for microstrip (Fig. 2), is written in Fortran IV and is compatible with the CDC 3800 digital computer system. With minor modifications this program should also be suitable for use with the CDC 6000-series computer systems, with the IBM 360 or 370 systems, and with any other computer having comparable storage capability and compatibility with the Fortran IV language. The core storage necessary for executing this program on the CDC 3800 system at the NRL Research Computation Center is 42,000 words. Although the total core storage of this computer is now 75,536 words, the maximum storage available for a single array (without resorting to special array-handling techniques) is 32,768 words. It is this system requirement that constrains the largest array in MS to be 181 × 181 or 32,761 words.

Furthermore, because of the standard loading procedures employed by the CDC 3800 system at NRL, the following scheme for handling large arrays was incorporated into MS. The two relatively large arrays in MS are array A, 181 × 181, and array A1, 132 × 132. These arrays appear in different subroutines and are used consecutively. For these reasons

the arrays A and A1 are placed in a block of COMMON storage labeled HELP. During the loading of the program, HELP is loaded into a storage bank with 32,768 storage addresses available to accommodate array A. This is done using a BANK card. In addition another BANK card forces main program MS and subroutine MSCUPF into a different storage bank. The storage required to operate using A and A1 is minimized by forcing them to share the same storage locations by means of the statement EQUIVALENCE (A, A1). These techniques are necessary to properly load MS into the CDC 3800 system at NRL.

The length of time necessary for program execution is approximately 9 minutes for each configuration to be analyzed.

Input Information

Each configuration of microstrip to be analyzed by program MS is characterized by specifying the cross section in terms of the quantities W, H, L, ϵ_1, $\tan \delta_1$, and σ (Fig. 2). Accordingly, the first data card for any execution of MS contains the quantity NSETS. This is an integer specifying the number of configurations to be analyzed. NSETS is punched on the first data card according to the format I10.

Each configuration included in NSETS is specified on a separate data card located consecutively behind the first data card. Each card lists the following six quantities punched according to the format F10.6:

W = width of the conducting strip in mils (1 mil = 25.4 μm);

H = height of the conducting strip above the ground plane (the substrate thickness) in mils;

L = substrate width in inches, which is a variable quantity to allow for narrow substrate widths, with a value L = 1.0 inch (2.54 cm) having been adequate to eliminate effects due to finite substrate width;

ER – relative permittivity of the substrate material;

TAND1 = loss tangent of the substrate dielectric ($\times 10^4$);

SIGMA = dc Conductivity of the metallization in mhos per meter ($\times 10^{-7}$).

The number of cards behind the first data card, each containing sets of these six parameters, should equal the value read in for NSETS.

Output Information

The first line of output is invariant for each configuration and indicates that the program performs computations taking the strip conductor to have a metallization thickness

of 6.35 μm. The next line is skipped, and the third line lists the preceding six input parameters for the configuration to be analyzed. After skipping another line, the following quantities are tabulated from left to right with identifying labels printed directly above computed values.

Z0(OHMS) = characteristic impedance of the transmission line in ohms;

V(M/SEC) = phase velocity in meters per second;

ALPHAC(DB/M)/SQRT(F) = conductor-loss coefficient in dB per meter divided by the square root of the frequency in hertz;

ALPHAD(DB/M)/F = dielectric-loss coefficient in dB per meter divided by the frequency in hertz.

For program executions in which more than one configuration is to be analyzed (NSETS > 1), an output block similar to that described is printed for each configuration. Four blank lines separate the output information blocks for each configuration.

Program CPW (Coplanar Waveguide)

Program CPW is written in Fortran IV and is compatible with the CDC 6700 computer system. With minor modifications this program should also be suitable for use with the IBM 360 or 370 systems and with any other computer having comparable storage capability and compatibility with the Fortran IV language. The storage necessary for executing this program on the CDC 6700 computer at the Naval Ship Research and Development Center (NSRDC) is 86,143 words. The length of time necessary for program execution is approximately 15 minutes per configuration.

Input Information

Each configuration of coplanar waveguide to be analyzed by program CPW is characterized by specifying the cross section in terms of the geometry shown in Fig. 5. Accordingly the first data card for any execution of CPW contains the quantity NSETS, specified in the I10 format. This is an integer specifying the number of configurations to be analyzed.

Each configuration included in NSETS is specified on a separate data card located consecutively behind the first data card. Each card lists the following quantities punched according to the format F10.6:

W = width of the conducting strip in mils (1 mil = 25.4 μm);

S = spacing between the strip edge and grounded conductors in mils;

H = thickness of the substrate in mils;

L = substrate width in inches, 1.0 inch (2.54 cm) being adequate to eliminate effects of finite width;

ER = relative permittivity of the substrate material;

TAND1 = loss tangent of the substrate dielectric ($\times 10^4$);

SIGMA = dc conductivity of the metallization in mhos per meter ($\times 10^{-7}$).

The number of cards behind the first data card should equal the value read in for NSETS.

Output Information

The output information provided by CPW is similar in format to that for program MS. The first line of output for each configuration indicates the metallization thickness. After a line is skipped, the input parameters are listed in a line of output. After another line is skipped, the quantities Z0(OHMS), V(M/SEC), ALPHAC(DB/M)/SQRT(F), and ALPHAD (DB/M)/F are printed in a manner similar to that described for program MS. For program executions in which more than one configuration is to be analyzed (NSETS > 1), an output block similar to that already described is printed for each configuration.

Program IM (Inverted Microstrip)

Program IM is written in Fortran IV and is compatible with the CDC 6700 computer system. With minor modifications this program should also be suitable for use with the IBM 360 or 370 systems and with any other computers having comparable storage capability and compatibility with the Fortran IV language. The storage necessary for executing this program on the CDC 6700 computer at the Naval Ship Research and Development Center is 78,698 words. The length of time necessary for program execution is approximately 12 minutes per configuration.

Input Information

Each configuration of inverted microstrip to be analyzed by program IM is characterized by specifying the cross section in terms of the geometry shown in Fig. 8. Accordingly the first data card for any execution of IM contains the quantity NSETS, specified in the I10 format. This is an integer specifying the number of configurations to be analyzed.

Each configuration included in NSETS is specified on a separate data card located consecutively behind the first data card. Each card requires the following quantities punched according to the format F10.6:

W = width of the conducting strip in mils (1 mil = 25.4 μm);

H = height of the strip above the ground plane in mils;

T = substrate thickness in mils;

L = substrate width in inches, 1.0 inch (2.54 cm) being adequate to eliminate effects due to finite width;

ER = relative permittivity of the substrate material;

TAND1 = loss tangent of the substrate dielectric ($\times 10^4$);

SIGMA = dc conductivity of the metallization in mhos per meter ($\times 10^{-7}$).

The number of cards behind the first data card should equal the value read in for NSETS.

Output Information

The output information provided by program IM appears in the same format as that described for programs MS and CPW.

Program TIM (Trapped Inverted Microstrip)

Program TIM is written in Fortran IV and is compatible with the CDC 3800 computer system. With minor modifications this program should be suitable for use with the CDC 6000-series computer systems, the IBM 360 or 370 systems, and any other computer having comparable storage capability and compatibility with the Fortran IV language. The core storage necessary for executing the program on the CDC 3800 system at the NRL Research Computation Center is 42,000 words. Although the total core storage of this computer is now 75,536 words, the maximum storage available for a single array (without resorting to special array — handling techniques) is 32,768 words. It is this system feature that constrains the largest array in TIM to be 181 × 181 or 32,761 words. To properly load this program into the computer, a procedure similar to the one described for program MS is used.

The length of time necessary for program execution is approximately 7.8 minutes per configuration.

Input Information

Each configuration of trapped inverted microstrip to be analyzed by program TIM is characterized by specifying the cross section in terms of the geometry shown in Fig. 12. Accordingly the first data card for any execution of TIM contains the quantity NSETS, specified in the I10 format. This is an integer specifying the number of configurations to be analyzed.

Each configuration included in NSETS is specified on a separate data card located consecutively behind the first data card. Each card lists the following quantities, punched according to the format F10.6:

W = width of the conducting strip in mils (1 mil = 25.4 μm);

S = spacing between the strip edge and the channel upper corner in mils;

H = height of the strip above the channel bottom in mils;

T = substrate thickness in mils;

L = substrate (and support block) width in inches, 1.0 inch (2.54 cm) being adequate to eliminate effects of substrate and support block finite widths;

ER = relative permittivity of the substrate material;

TAND1 = loss tangent of the substrate dielectric ($\times 10^4$);

SIGMA = dc conductivity of the metallization specified in mhos per meter ($\times 10^{-7}$).

The number of cards behind the first data card should equal the value read in for NSETS.

Output Information

The output information provided by program TIM appears in the same format as that described for programs MS and CPW.

Program LEMDOC (Lossy Edge-Coupled-Microstrip Dielectric-Overlay Coupler)

Program LEMDOC is written in Fortran IV and is compatible with the CDC 3800 digital computer. With minor modifications this program should also be compatible with the CDC 6000-series computers, the IBM 360 or 370 systems, or any other computer which has similar storage capability and compatibility with the Fortran IV language. The core storage necessary for executing this program on the CDC 3800 system at the NRL Research Computation Center is about 42,000 words. Although the total storage of this computer is now 75,536 words, the maximum storage available for a single array (without resorting to special array-handling techniques) is 32,768 words. It is this system requirement that constrains the largest array in LEMDOC to be 181 × 181 or 32,761. Special techniques required for the loading of this program are similar to those described for program MS.

The length of time necessary for program execution is approximately 15 minutes per configuration.

Input Information

Each configuration of edge-coupled microstrip with a dielectric overlay to be analyzed by program LEMDOC can be characterized by specifying the cross section in terms of the

geometry shown in Fig. 12. Accordingly the first data card for any execution of LFMDOC contains the quantity NSETS, an integer specifying the number of configurations to be analyzed. NSETS is punched on the first data cards in the I10 format.

Each configuration included in NSETS is specified on a separate data card located consecutively behind the first data card. Each card requires the following quantities, punched according to the F10.6 format:

W = conducting strip width in mils (1 mil = 25.4 μm);

S = spacing between the two conducting strips in mils;

T = thickness of the dielectric overlay in mils;

H = height of the strips above the ground plane in mils;

L = width of the supporting substrate in inches, L = 1.0 inch (2.54 cm) being adequate to eliminate effects due to finite width;

ER = relative permittivity of the substrate and overlay dielectric material;

TAND1 = loss tangent of the substrate and overlay dielectric ($\times 10^4$);

SIGMA = dc conductivity of the metallization in mhos per meter ($\times 10^{-7}$).

The number of cards behind the first data card should equal the value of NSETS.

Output Information

An output information block is printed for each consecutive input data block. The first line of output stipulates the thickness of the metallization. After a line is skipped, the input parameters are listed from left to right. After another line is skipped, the following output quantities are tabulated directly under an appropriate label:

C(DB) = computed value of midband coupling in dB;

ZO(OHMS) = coupled line characteristic impedance in ohms;

ZOO(OHMS) = odd-mode impedance in ohms;

ZOE(OHMS) = even-mode impedance in ohms;

VO(M/SEC) = odd-mode phase velocity in meters per second;

VE(M/SEC) = even-mode phase velocity in meters/second;

VAVG(M/SEC) = average of VO(M/SEC) and VE(M/SEC) in meters per second;

ALFACO(DB/M)/SQRT(F) = odd-mode conductor-loss coefficient in dB per meter divided by the square root of the frequency in hertz;

ALFACE(DB/M)/SQRT(F) = the even-mode conductor-loss coefficient in dB per meter divided by the square root of the frequency in hertz;

ALFADO(DB/M)/F = odd-mode dielectric-loss coefficient in dB per meter divided by the frequency in hertz;

ALFADE(DB/M)/F = even-mode dielectric-loss coefficient in dB per meter divided by the frequency in hertz.

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REFERENCES

1. C.P. Wen, "Coplanar Waveguide: A Surface Strip Transmission Line Suitable for Nonreciprocal Gyromagnetic Applications," IEEE Trans. Micr. Th. Techs. MTT-17 (No. 12), 1087-1096 (Dec. 1969).

2. M.V. Schneider, "Microstrip Lines for Microwave Integrated Circuits," Bell System Tech. J. 48 (No. 5), 1421-1444 (May-June 1969).

3. B.E. Spielman, "Analysis of Electrical Characteristics of Edge-Coupled Microstrip Lines with a Dielectric Overlay," NRL Report 7810, Oct. 25, 1974.

4. R.F. Harrington, et al., "Computation of Laplacian Potentials by an Equivalent Source Method," Proc. IEE 116 (No. 10), 1715-1720 (Oct. 1969).

5. R.F. Harrington, *Time-Harmonic Electromagnetic Fields*, McGraw-Hill, New York, 1961, pp. 61-67.

6. J.D. Jackson, *Classical Electrodynamics*, Wiley, New York, 1966, pp. 236-240.

7. L. Young, (editor), *Advances in Microwaves*, Vol. 1, Academic Press, New York, 1966, pp. 115-209.

8. C.D. Hodgman, (editor), et al., *Handbook of Chemistry and Physics*, Chemical Rubber Publishing Co., Cleveland, Ohio, 1961, p. 2628.

9. R.A. Pucel et al., "Losses in Microstrip," IEEE Trans. Micr. Th. Techs. MTT-16 (No. 6), 312-350 (June 1968).

10. M. Caulton et al., "Measurement of the Properties of Microstrip Transmission Lines for Microwave Integrated Circuits," RCA Review 27 (No. 3), 377-391 (Sept. 1966).

11. M.V. Schneider, "Dielectric Loss in Integrated Microwave Circuits," Bell System Tech. J. 48, 2325-2332 (Sept. 1969).

12. T.L. Simpson and B. Tseng, "Dielectric Loss in Microstrip Lines," IEEE Trans. Micr. Th. Techs. MTT-24 (No. 2), 106-108 (Feb. 1976).

13. J. McDade and D. Stockman, "Microwave Integrated Circuit Techniques," Tech. Rept. AFAL-TR-73-234, AFAL/TEM, Wright Patterson AFB, Ohio 45433, May 1973.

14. B. Sheleg and B.E. Spielman, "Broad Band Directional Couplers Using Microstrip with Dielectric Overlays," IEEE Trans. Micr. Th. Techs. MTT-22 (No. 12), Part II, 1216-1220 (Dec. 1974).

Appendix A

LISTING OF PROGRAM MS

```
      PROGRAM MS
      REAL L
      READ 1, NSETS
    1 FORMAT(I10)
      DO 2 MN=1,NSETS
      READ 3, W,H,L,ER,TAND1,SIGMA
    3 FORMAT(6F10.6)
      TAND1=TAND1*1.E-04
      SIGMA=SIGMA*1.E+07
      PRINT 8
    8 FORMAT(1H ,////1H ,*STRIP METALIZATION THICKNESS = 250 MICROINCHES
     1*)
      PRINT 4, W,H,L,ER,TAND1,SIGMA
    4 FORMAT(1H0,2HW=,F5.1,3X,2HH=,F5.1,3X,2HL=,
     1F10.6,3X,3HER=,F10.6,3X,6HTAND1=,F10.6,3X,6HSIGMA=,F10.6)
      FACTOR=(1.0E-05)*(1./0.393700)
      W=W*FACTOR
      H=H*FACTOR
      L=L*FACTOR*(1.E+03)
      E1=ER
      E2=ER+0.01*ER
      CALL MSF(W,H,L,E1,SIGMA,C,PDDSQ)
      PDDSQF=PDDSQ
      CALL MSF(W,H,L,E2,SIGMA,CP,PDDSQ)
      CALL MSE(W,H,L,CE)
      A=C*CE
      B=CE/C
      Z0=1./((3.0E+08)*SQRT(A))
      V=3.0E+08*SQRT(B)
      ARG1=B
      ARG2=CE/CP
      ER1=1./ARG1
      ER2=1./ARG2
      FACTC=10./2.3
      FACTD=27.3/3.E+08
      AC=FACTC*PDDSQF*ER1*Z0
      AD=FACTD*(E1/SQRT(ER1))*((ER2-ER1)/(E2-E1))*TAND1
      PRINT 5
    5 FORMAT(1H0,1X,8HZ0(OHMS),5X,8HV(M/SEC),5X,
     120HALPHAC(DB/M)/SQRT(F),5X,14HALPHAD(DB/M)/F)
      PRINT 6, Z0,V,AC,AD
    6 FORMAT(1H ,2X,F6.2,4X,E10.3,8X,E10.3,11X,E10.3)
    2 CONTINUE
      STOP
      END
      SUBROUTINE MSF(W,H,L,ER,SIGMA,CHRG,PDDSQF)
      DIMENSION N(4),X(91,4),Y(91,4),ALPHA(91,4),BETA1(91,4),
     1BETA2(91,4),GAMMA(91,4),CH(91,4),SCH(4),TSCD(91,4,4),A(181,181),
     2A1(134,134)
      REAL L
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
      NO=4
      N(1)=43
      N(2)=91
      N(3)=25
      N(4)=25
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      AAA=3.14159*4.E-07*3.14159/SIGMA
```

```
      RDSQF=SQRT(AAA)
      FACTN=4.*3.14159*3.14159*RDSQF*8.85E-12
      FACTD=4.*3.14159*1.E-07
      FACTL=FACTN/FACTD
      FACTOR=(1.0E-05)*(1./0.393700)
      DELW=W/20.
      E=ER*E0
      X(1,1)=-W/2.
      Y(1,1)=H
      DO 26 I=1,20
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
   26 Y(IP1,1)=H
      X(22,1)=X(21,1)
      Y(22,1)=0.25*FACTOR+H
      DO 27 I=22,41
      IP1=I+1
      X(IP1,1)=X(I,1)-DELW
   27 Y(IP1,1)=Y(I,1)
      X(43,1)=X(1,1)
      Y(43,1)=Y(1,1)
      DELL=L/44.
      X(1,2)=-L/2.
      Y(1,2)=0.
      DO 28 I=1,44
      IP1=I+1
      X(IP1,2)=X(I,2)+DELL
   28 Y(IP1,2)=0.
      X(46,2)=X(45,2)
      Y(46,2)=-0.25*FACTOR
      DO 60 I=46,89
      IP1=I+1
      X(IP1,2)=X(I,2)-DELL
   60 Y(IP1,2)=Y(I,2)
      X(91,2)=X(1,2)
      Y(91,2)=Y(1,2)
      DELH=H/10.
      DELD=(L/2.-W/2.)/14.
      X(1,3)=X(1,2)
      Y(1,3)=Y(1,2)
      X(1,4)=-X(1,3)
      Y(1,4)=Y(1,3)
      DO 61 I=1,10
      IP1=I+1
      X(IP1,3)=X(I,3)
      Y(IP1,3)=Y(I,3)+DELH
      X(IP1,4)=X(I,4)
   61 Y(IP1,4)=Y(IP1,3)
      DO 62 I=11,24
      IP1=I+1
      X(IP1,3)=X(I,3)+DELD
      Y(IP1,3)=Y(I,3)
      X(IP1,4)=X(I,4)-DELD
   62 Y(IP1,4)=Y(IP1,3)
      DO 63 I=1,42
      ALPHA(I,1)=1.
      BETA1(I,1)=0.
      BETA2(I,1)=0.
   63 GAMMA(I,1)=1.
```

```
      DO 64 I=1,90
      ALPHA(I,2)=1.
      BETA1(I,2)=0.
      BETA2(I,2)=0.
   64 GAMMA(I,2)=0.
      DO 65 I=1,24
      ALPHA(I,3)=0.
      BETA1(I,3)=E
      BETA2(I,3)=E0
      GAMMA(I,3)=0.
      ALPHA(I,4)=0.
      BETA1(I,4)=E0
      BETA2(I,4)=E
   65 GAMMA(I,4)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=91
      R=1.0E+05
      NAXDIM=181
      NAYDIM=181
      CALL LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DELF=W/20.
      DELE=0.25*FACTOR
      FF1=6.2831852*E*DELF
      FF2=6.2831852*E0*DELF
      FF4=6.2831852*E0*DELE
      CHRG=0.
      DO 50 I=1,20
   50 CHRG=CHRG+FF1*CH(I,1)
      DO 51 I=22,41
   51 CHRG=CHRG+FF2*CH(I,1)
      CHRG=CHRG+FF4*(CH(21,1)+CH(42,1))
      DO 1064 J=1,NO
      IF(ALPHA(1,J).NE.1.) GO TO 64
      NJ1=N(J)-1
      DO 1065 I=1,NJ1
      IP1=I+1
      SEGX=X(IP1,J)-X(I,J)
      SEGY=Y(IP1,J)-Y(I,J)
      SEGARG=SEGX*SEGX+SEGY*SEGY
      SEGLEN=SQRT(SEGARG)
 1065 SUM=SUM+CH(I,J)*CH(I,J)*SEGLEN
 1064 CONTINUE
      PDDSQF=FACTL*SUM
   54 CONTINUE
      RETURN
      END
      SUBROUTINE LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A(181,181),SCH(NO),B(180),A1(134,134)
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
```

```
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
```

```
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
   16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
   17 RETURN
      END
      SUBROUTINE MSE(W,H,L,CE)
      DIMENSION N(2),X(91,2),Y(91,2),ALPHA(91,2),BETA1(91,2),
     1BETA2(91,2),GAMMA(91,2),CH(91,2),SCH(2),TSCD(91,2,4),A(181,181),
     2A1(134,134)
      REAL L
      COMMON/HELP/A,A1
```

```
      EQUIVALENCE(A,A1)
      NO=2
      N(1)=43
      N(2)=91
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      FACTOR=(1.0E-05)*(1./0.393700)
      DELW=W/20.
      X(1,1)=-W/2.
      Y(1,1)=H
      DO 26 I=1,20
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
   26 Y(IP1,1)=H
      X(22,1)=X(21,1)
      Y(22,1)=0.25*FACTOR+H
      DO 27 I=22,41
      IP1=I+1
      X(IP1,1)=X(I,1)-DELW
   27 Y(IP1,1)=Y(I,1)
      X(43,1)=X(1,1)
      Y(43,1)=Y(1,1)
      DELL=L/44.
      X(1,2)=-L/2.
      Y(1,2)=0.
      DO 28 I=1,44
      IP1=I+1
      X(IP1,2)=X(I,2)+DELL
   28 Y(IP1,2)=0.
      X(46,2)=X(45,2)
      Y(46,2)=-0.25*FACTOR
      DO 60 I=46,89
      IP1=I+1
      X(IP1,2)=X(I,2)-DELL
   60 Y(IP1,2)=Y(I,2)
      X(91,2)=X(1,2)
      Y(91,2)=Y(1,2)
      DO 63 I=1,42
      ALPHA(I,1)=1.
      BETA1(I,1)=0.
      BETA2(I,1)=0.
   63 GAMMA(I,1)=1.
      DO 64 I=1,90
      ALPHA(I,2)=1.
      BETA1(I,2)=0.
      BETA2(I,2)=0.
   64 GAMMA(I,2)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=91
      R=1.0E+05
      NAXDIM=134
      NAYDIM=134
      CALL LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      CE=6.2831852*E0*SCH(1)
```

```
      RETURN
      END
      SUBROUTINE LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A(181,181),SCH(NO),B(132),A1(134,134)
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      +F(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
```

```
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
```

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```
 16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
 17 RETURN
    END
    SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
    DIMENSION S(1),D(1)
    NI=N-I
    IF(MODE-1) 100,100,120
100 IJ=I*J+1
    NM=N*J+1
    DO 110 K=1,J
    NM=NM-NI
    DO 110 L=1,I
    IJ=IJ-1
    NM=NM-1
110 D(NM)=S(IJ)
    GO TO 140
120 IJ=0
    NM=0
    DO 130 K=1,J
    DO 125 L=1,I
    IJ=IJ+1
    NM=NM+1
125 S(IJ)=D(NM)
130 NM=NM+NI
140 RETURN
    END
    SUBROUTINE SIMQ(A,B,N,KS)
    DIMENSION A(1),B(1)
    TOL=0.0
    KS=0
    JJ=-N
    DO 65 J=1,N
    JY=J+1
    JJ=JJ+N+1
    BIGA=0.
    IT=JJ-J
    DO 30 I=J,N
    IJ=IT+I
    IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
 20 BIGA=A(IJ)
    IMAX=I
 30 CONTINUE
    IF(ABS(BIGA)-TOL) 35,35,40
 35 KS=1
    RETURN
 40 I1=J+N*(J-2)
    IT=IMAX-J
    DO 50 K=J,N
    I1=I1+N
    I2=I1+IT
    SAVE=A(I1)
    A(I1)=A(I2)
    A(I2)=SAVE
 50 A(I1)=A(I1)/BIGA
    SAVE=B(IMAX)
    B(IMAX)=B(J)
    B(J)=SAVE/BIGA
    IF(J-N) 55,70,55
 55 IQS=N*(J-1)
```

```
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

Appendix B

LISTING OF PROGRAM CPW

```
      PROGRAM CPW(INPUT,OUTPUT)
      REAL L
      READ 1, NSETS
    1 FORMAT(I10)
      DO 2 MN=1,NSETS
      READ 3, W,S,H,L,ER,TAND1,SIGMA
    3 FORMAT(7F10.6)
      TAND1=TAND1*1.0E-04
      SIGMA=SIGMA*1.0E+07
      PRINT 8
    8 FORMAT(1H ,////1H ,*STRIP METALIZATION THICKNESS = 230 MICROINCHES
     1*)
      PRINT 4, W,S,H,L,ER,TAND1,SIGMA
    4 FORMAT(1H0,2HW=,F5.1,3X,2HS=,F5.1,3X,2HH=,F5.1,3X,2HL=,
     1F10.6,3X,3HER=,F10.6,3X,6HTAND1=,E10.3,3X,6HSIGMA=,E10.3)
      FACTOR=(1.0E-05)*(1./0.393700)
      W=W*FACTOR
      S=S*FACTOR
      H=H*FACTOR
      L=L*FACTOR*(1.E+03)
      E1=ER
      E2=ER+0.01*ER
      CALL CPWF(W,S,H,L,E1,SIGMA,C,PDDSQ)
      PDDSQF=PDDSQ
      CALL CPWF(W,S,H,L,E2,SIGMA,CP,PDDSQ)
      CALL CPWE(W,S,H,L,CE)
      A=C*CE
      B=CE/C
      Z0=1./((3.E+08)*SQRT(A))
      V=3.E+08*SQRT(B)
      ARG1=B
      ARG2=CE/CP
      ER1=1./ARG1
      ER2=1./ARG2
      FACTC=10./2.3
      FACTD=27.3/3.E+08
      AC=FACTC*PDDSQF*ER1*Z0
      AD=FACTD*(E1/SQRT(ER1))*((ER2-ER1)/(E2-E1))*TAND1
      PRINT 5
    5 FORMAT(1H0,1X,8HZ0(OHMS),5X,8HV(M/SEC),5X,
     120HALPHAC(DB/M)/SQRT(F),5X,14HALPHAD(DB/M)/F)
      PRINT 6,Z0,V,AC,AD
    6 FORMAT(1H ,2X,F6.2,4X,E10.3,8X,E10.3,11X,E10.3)
    2 CONTINUE
      STOP
      END
      SUBROUTINE CPWF(W,S,H,L,ER,SIGMA,CHRG,PDDSQF)
      DIMENSION N(6),X(59,6),Y(59,6),ALPHA(59,6),BETA1(59,6),
     1BETA2(59,6),GAMMA(59,6),CH(59,6),SCH(6),TSCD(59,6,4),A(252,252)
      REAL L
      COMMON/HELP/Z(252,252)
      EQUIVALENCE(Z,A)
      NO=6
      N(1)=41
      N(2)=59
      N(3)=59
      N(4)=21
      N(5)=21
      N(6)=51
```

```
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      AAA=3.14159*4.E-07*3.14159/SIGMA
      RDSQF=SQRT(AAA)
      FACTN=4.*3.14159*3.14159*RDSQF*8.85E-12
      FACTD=4.*3.14159*1.E-07
      FACTL=FACTN/FACTD
      FACTOR=(1.0E-05)*(1./0.393700)
      DELW=W/19.
      E=ER*E0
      X(1,1)=-W/2.
      Y(1,1)=-0.25*FACTOR
      DO 26 I=1,19
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
   26 Y(IP1,1)=Y(1,1)
      X(21,1)=X(20,1)
      Y(21,1)=0.
      DO 27 I=21,39
      IP1=I+1
      X(IP1,1)=X(I,1)-DELW
   27 Y(IP1,1)=Y(I,1)
      X(41,1)=X(1,1)
      Y(41,1)=Y(1,1)
      DELG=(L/2.-W/2.-S)/28.
      X(1,2)=-L/2.
      Y(1,2)=0.
      X(30,2)=-(W/2.)-S
      Y(30,2)=-0.25*FACTOR
      X(1,3)=L/2.
      Y(1,3)=0.
      X(30,3)=(W/2.)+S
      Y(30,3)=-0.25*FACTOR
      DO 28 I=1,28
      IP1=I+1
      IP30=I+30
      IP30M1=IP30-1
      X(IP1,2)=X(I,2)+DELG
      Y(IP1,2)=0.
      X(IP30,2)=X(IP30M1,2)-DELG
      Y(IP30,2)=Y(30,2)
      X(IP1,3)=X(I,3)-DELG
      Y(IP1,3)=0.
      X(IP30,3)=X(IP30M1,3)+DELG
   28 Y(IP30,3)=Y(30,3)
      X(59,2)=X(1,2)
      Y(59,2)=Y(1,2)
      X(59,3)=X(1,3)
      Y(59,3)=Y(1,3)
      DELS=S/20.
      X(1,4)=-W/2.-S
      Y(1,4)=0.
      X(1,5)=W/2.+S
      Y(1,5)=0.
      DO 29 I=1,20
      IP1=I+1
      X(IP1,4)=X(I,4)+DELS
      Y(IP1,4)=0.
      X(IP1,5)=X(I,5)-DELS
   29 Y(IP1,5)=0.
```

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```
      DELH=H/2.
      DELL=L/46.
      X(1,6)=-L/2.
      Y(1,6)=0.
      X(49,6)=L/2.
      Y(49,6)=H
      DO 30 I=1,2
      IP1=I+1
      IP49=I+49
      IP49M1=IP49-1
      X(IP1,6)=X(I,6)
      Y(IP1,6)=Y(I,6)+DELH
      X(IP49,6)=X(IP49M1,6)
   30 Y(IP49,6)=Y(IP49M1,6)-DELH
      DO 31 I=1,46
      IP3=I+3
      IP3M1=IP3-1
      X(IP3,6)=X(IP3M1,6)+DELL
   31 Y(IP3,6)=H
      DO 60 I=1,40
      ALPHA(I,1)=1.
      BETA1(I,1)=0.
      BETA2(I,1)=0.
   60 GAMMA(I,1)=1.
      DO 61 I=1,58
      ALPHA(I,2)=1.
      ALPHA(I,3)=1.
      BETA1(I,2)=0.
      BETA1(I,3)=0.
      BETA2(I,2)=0.
      BETA2(I,3)=0.
      GAMMA(I,2)=0.
   61 GAMMA(I,3)=0.
      DO 62 I=1,20
      ALPHA(I,4)=0.
      ALPHA(I,5)=0.
      BETA1(I,4)=E0
      BETA1(I,5)=E
      BETA2(I,4)=E
      BETA2(I,5)=E0
      GAMMA(I,4)=0.
   62 GAMMA(I,5)=0.
      DO 63 I=1,50
      ALPHA(I,6)=0.
      BETA1(I,6)=E
      BETA2(I,6)=E0
   63 GAMMA(I,6)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=59
      R=1.0E+05
      NAXDIM=252
      NAYDIM=252
      CALL LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
```

```
      DELF=W/19.
      DELE=0.25*FACTOR
      FF1=6.2831852*E*DELF
      FF2=6.2831852*E0*DELE
      FF4=6.2831852*E0*DELF
      CHRG=0.
      DO 50 I=1,19
   50 CHRG=CHRG+FF4*CH(I,1)
      DO 51 I=21,39
   51 CHRG=CHRG+FF1*CH(I,1)
      CHRG=CHRG+FF2*(CH(20,1)+CH(40,1))
      SUM=0.
      DO 64 J=1,NO
      IF(ALPHA(1,J).NE.1.) GO TO 64
      NJ1=N(J)-1
      DO 65 I=1,NJ1
      IP1=I+1
      SEGX=X(IP1,J)-X(I,J)
      SEGY=Y(IP1,J)-Y(I,J)
      SEGARG=SEGX*SEGX+SEGY*SEGY
      SEGLEN=SQRT(SEGARG)
   65 SUM=SUM+CH(I,J)*CH(I,J)*SEGLEN
   64 CONTINUE
      PDDSQF=FACTL*SUM
   54 CONTINUE
      RETURN
      END
      SUBROUTINE LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A(252,252),SCH(NO),B(246)
      COMMON/HELP/Z(252,252)
      EQUIVALENCE(Z,A)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
```

```
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13
```

```
12 DY=0.
   GO TO 14
13 DY=(YMAX-YMIN)/FLOAT(NY-1)
14 DO 16 II=1,NX
   XJ=XMIN+FLOAT(II-1)*DX
   DO 16 JJ=1,NY
   YJ=YMIN+FLOAT(JJ-1)*DY
   A(II,JJ)=0.
   DO 16 LI=1,NO
   NN=N(LI)-1
   DO 16 I=1,NN
   X1=XJ-X(I,LI)
   X2=XJ-X(I+1,LI)
   Y1=YJ-Y(I,LI)
   Y2=YJ-Y(I+1,LI)
   R1=X1*X1+Y1*Y1
   R2=X2*X2+Y2*Y2
   IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
   YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
   XT=X1*X2+Y1*Y2
   TETA=ATAN2(YT,XT)
   S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
  1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
   GO TO 16
15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
17 RETURN
   END
   SUBROUTINE CPWE(W,S,H,L,CH1)
   DIMENSION N(3),X(70,3),Y(70,3),ALPHA(70,3),BETA1(70,3),
  1BETA2(70,3),GAMMA(70,3),CH(70,3),SCH(3),TSCD(70,3,4),A1(181,181)
   REAL L
   COMMON/HELP/Z(252,252)
   EQUIVALENCE(Z,A1)
   NO=3
   N(1)=41
   N(2)=70
   N(3)=70
   E0=1./(4.*3.14159*2.99776*2.99776E+09)
   FACTOR=(1.0E-05)*(1./0.393700)
   DELW=W/40.
   DELG=(L/2.-W/2.-S)/69.
   X(1,1)=-W/2.
   Y(1,1)=0.
   DO 26 I=1,40
   IP1=I+1
   X(IP1,1)=X(I,1)+DELW
   Y(IP1,1)=Y(I,1)
   ALPHA(I,1)=1.
   BETA1(I,1)=0.
   BETA2(I,1)=0.
26 GAMMA(I,1)=1.
   X(1,2)=-L/2.
   Y(1,2)=0.
   X(1,3)=L/2.
   Y(1,3)=0.
   DO 27 I=1,69
   IP1=I+1
   X(IP1,2)=X(I,2)+DELG
```

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```
      Y(IP1,2)=Y(I,2)
      X(IP1,3)=X(I,3)-DELG
      Y(IP1,3)=Y(I,3)
      ALPHA(I,2)=1.
      BETA1(I,2)=0.
      BETA2(I,2)=0.
      GAMMA(I,2)=0.
      ALPHA(I,3)=1.
      BETA1(I,3)=0.
      BETA2(I,3)=0.
   27 GAMMA(I,3)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=70
      R=1.0E+05
      NAXDIM=181
      NAYDIM=181
      CALL LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      CH1=SCH(1)*6.2831852*E0
      RETURN
      END
      SUBROUTINE LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A1(181,181),SCH(NO),B(180)
      COMMON/HELP/Z(252,252)
      EQUIVALENCE(Z,A1)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
```

```
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
```

```
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
   16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
   17 RETURN
      END
      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100,100,120
  100 IJ=I*J+1
      NM=N*J+1
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
  130 NM=NM+NI
  140 RETURN
      END
      SUBROUTINE SIMQ(A,B,N,KS)
      DIMENSION A(1),B(1)
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0.
```

```
      IT=JJ-J
      DO 30 I=J,N
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

Appendix C

LISTING OF PROGRAM IM

```
      PROGRAM IM(INPUT,OUTPUT)
      REAL L
      READ 1, NSETS
    1 FORMAT(I10)
      DO 2 MN=1,NSETS
      READ 3, W,H,T,L,ER,TAND1,SIGMA
    3 FORMAT(7F10.6)
      TAND1=TAND1*1.0E-04
      SIGMA=SIGMA*1.0E+07
      PRINT 8
    8 FORMAT(1H ,////1H ,*STRIP METALIZATION THICKNESS = 250 MICROINCHES
     1*)
      PRINT 4, W,H,T,L,ER,TAND1,SIGMA
    4 FORMAT(1H0,2HW=,F5.1,3X,2HH=,F5.1,3X,2HT=,F5.1,3X,2HL=,
     1F10.6,3X,3HER=,F10.6,3X,6HTAND1=,E10.3,3X,6HSIGMA=,E10.3)
      FACTOR=(1.0E-05)*(1./0.393700)
      W=W*FACTOR
      H=H*FACTOR
      T=T*FACTOR
      L=L*FACTOR*(1.E+03)
      E1=ER
      E2=ER+0.01*ER
      CALL SUSMSF(W,H,T,L,E1,SIGMA,C,PDDSQ)
      PDDSQF=PDDSQ
      CALL SUSMSF(W,H,T,L,E2,SIGMA,CP,PDDSQ)
      CALL SUSMSE(W,H,T,L,CE)
      A=C*CE
      B=CE/C
      Z0=1./((3.E+08)*SQRT(A))
      V=3.E+08*SQRT(B)
      ARG1=B
      ARG2=CE/CP
      ER1=1./ARG1
      ER2=1./ARG2
      FACTC=10./2.3
      FACTD=27.3/3.E+08
      AC=FACTC*PDDSQF*ER1*Z0
      AD=FACTD*(E1/SQRT(ER1))*((ER2-ER1)/(E2-E1))*TAND1
      PRINT 5
    5 FORMAT(1H0,1X,8HZ0(OHMS),5X,8HV(M/SEC),5X,
     120HALPHAC(DB/M)/SQRT(F),5X,14HALPHAD(DB/M)/F)
      PRINT 6,Z0,V,AC,AD
    6 FORMAT(1H ,2X,F6.2,4X,E10.3,8X,E10.3,11X,E10.3)
    2 CONTINUE
      STOP
      END
      SUBROUTINE SUSMSF(W,H,T,L,ER,SIGMA,CHRG,PDDSQF)
      DIMENSION N(6),X(51,6),Y(51,6),ALPHA(51,6),BETA1(51,6),
     1BETA2(51,6),GAMMA(51,6),CH(51,6),SCH(6),TSCD(51,6,4),A(238,238)
      REAL L
      COMMON/HELP/Z(238,238)
      EQUIVALENCE(Z,A)
      NO=6
      N(1)=43
      N(2)=41
      N(3)=41
      N(4)=31
      N(5)=31
      N(6)=51
```

```
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      AAA=3.14159*4.E-07*3.14159/SIGMA
      RDSQF=SQRT(AAA)
      FACTN=4.*3.14159*3.14159*RDSQF*8.85E-12
      FACTD=4.*3.14159*1.E-07
      FACTL=FACTN/FACTD
      FACTOR=(1.0E-05)*(1./0.393700)
      DELW=W/20.
      E=ER*E0
      X(1,1)=-W/2.
      Y(1,1)=-0.25*FACTOR
      DO 26 I=1,20
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
   26 Y(IP1,1)=Y(1,1)
      X(22,1)=X(21,1)
      Y(22,1)=0.
      DO 27 I=22,41
      IP1=I+1
      X(IP1,1)=X(I,1)-DELW
   27 Y(IP1,1)=Y(I,1)
      X(43,1)=X(1,1)
      Y(43,1)=Y(1,1)
      DELG=(L/2.)/40.
      X(1,2)=-L/2.
      Y(1,2)=-S
      X(1,3)=L/2.
      Y(1,3)=-S
      DO 28 I=1,40
      IP1=I+1
      X(IP1,2)=X(I,2)+DELG
      Y(IP1,2)=-S
      X(IP1,3)=X(I,3)-DELG
   28 Y(IP1,3)=-S
      DELSUB=(L/2.-W/2.)/30.
      X(1,4)=-L/2.
      Y(1,4)=0.
      X(1,5)=L/2.
      Y(1,5)=0.
      DO 29 I=1,30
      IP1=I+1
      X(IP1,4)=X(I,4)+DELSUB
      Y(IP1,4)=0.
      X(IP1,5)=X(I,5)-DELSUB
   29 Y(IP1,5)=0.
      DELH=H/2.
      DELL=L/46.
      X(1,6)=-L/2.
      Y(1,6)=0.
      X(49,6)=L/2.
      Y(49,6)=H
      DO 30 I=1,2
      IP1=I+1
      IP49=I+49
      IP49M1=IP49-1
      X(IP1,6)=X(I,6)
      Y(IP1,6)=Y(I,6)+DELH
      X(IP49,6)=X(IP49M1,6)
   30 Y(IP49,6)=Y(IP49M1,6)-DELH
```

```
      DO 31 I=1,46
      IP3=I+3
      IP3M1=IP3-1
      X(IP3,6)=X(IP3M1,6)+DELL
   31 Y(IP3,6)=H
      DO 60 I=1,42
      ALPHA(I,1)=1.
      BETA1(I,1)=0.
      BETA2(I,1)=0.
   60 GAMMA(I,1)=1.
      DO 61 I=1,40
      ALPHA(I,2)=1.
      ALPHA(I,3)=1.
      BETA1(I,2)=0.
      BETA1(I,3)=0.
      BETA2(I,2)=0.
      BETA2(I,3)=0.
      GAMMA(I,2)=0.
   61 GAMMA(I,3)=0.
      DO 62 I=1,30
      ALPHA(I,4)=0.
      ALPHA(I,5)=0.
      BETA1(I,4)=E0
      BETA1(I,5)=E
      BETA2(I,4)=E
      BETA2(I,5)=E0
      GAMMA(I,4)=0.
   62 GAMMA(I,5)=0.
      DO 63 I=1,50
      ALPHA(I,6)=0.
      BETA1(I,6)=E
      BETA2(I,6)=E0
   63 GAMMA(I,6)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=51
      R=1.0E+05
      NAXDIM=238
      NAYDIM=238
      CALL LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DELF=W/20.
      DELE=0.25*FACTOR
      FF1=6.2831852*E*DELF
      FF2=6.2831852*E0*DELE
      FF4=6.2831852*E0*DELF
      CHRG=0.
      DO 50 I=1,20
   50 CHRG=CHRG+FF4*CH(I,1)
      DO 51 I=22,41
   51 CHRG=CHRG+FF1*CH(I,1)
      CHRG=CHRG+FF2*(CH(21,1)+CH(42,1))
      SUM=0.
      DO 64 J=1,NO
      IF(ALPHA(1,J).NE.1.) GO TO 64
```

```
      NJ1=N(J)-1
      DO 65 I=1,NJ1
      IP1=I+1
      SEGX=X(IP1,J)-X(I,J)
      SEGY=Y(IP1,J)-Y(I,J)
      SEGARG=SEGX*SEGX+SEGY*SEGY
      SEGLEN=SQRT(SEGARG)
   65 SUM=SUM+CH(I,J)*CH(I,J)*SEGLEN
   64 CONTINUE
      PDDSQF=FACTL*SUM
   54 CONTINUE
      RETURN
      END
      SUBROUTINE LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A(238,238),SCH(NO),B(237)
      COMMON/HELP/Z(238,238)
      EQUIVALENCE(Z,A)
      PI=3.1415925
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
```

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```
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
```

```
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
   16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
   17 RETURN
      END
      SUBROUTINE SUSMSE(W,H,T,L,CH1)
      DIMENSION N(3),X(70,3),Y(70,3),ALPHA(70,3),BETA1(70,3),
     1BETA2(70,3),GAMMA(70,3),CH(70,3),SCH(3),TSCD(70,3,4),A1(181,181)
      REAL L
      COMMON/HELP/Z(238,238)
      EQUIVALENCE(Z,A1)
      NO=3
      N(1)=41
      N(2)=70
      N(3)=70
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      FACTOR=(1.0E-05)*(1./0.393700)
      DELW=W/40.
      DELG=(L/2.)/69.
      X(1,1)=-W/2.
      Y(1,1)=0.
      DO 26 I=1,40
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
      Y(IP1,1)=Y(I,1)
      ALPHA(I,1)=1.
      BETA1(I,1)=0.
      BETA2(I,1)=0.
   26 GAMMA(I,1)=1.
      X(1,2)=-L/2.
      Y(1,2)=-S
      X(1,3)=L/2.
      Y(1,3)=-S
      DO 27 I=1,69
      IP1=I+1
      X(IP1,2)=X(I,2)+DELG
      Y(IP1,2)=Y(I,2)
      X(IP1,3)=X(I,3)-DELG
      Y(IP1,3)=Y(I,3)
      ALPHA(I,2)=1.
      BETA1(I,2)=0.
      BETA2(I,2)=0.
      GAMMA(I,2)=0.
      ALPHA(I,3)=1.
      BETA1(I,3)=0.
      BETA2(I,3)=0.
   27 GAMMA(I,3)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
```

```
      YMAX=0.
      NX=0
      NY=0
      IDIM=70
      R=1.0E+05
      NAXDIM=181
      NAYDIM=181
      CALL LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      CH1=SCH(1)*6.2831852*E0
      RETURN
      END
      SUBROUTINE LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1   TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A1(181,181),SCH(NO),B(180)
      COMMON/HELP/Z(238,238)
      EQUIVALENCE(Z,A1)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
```

```
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,",M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
```

COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION

```
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR),
   16 A(II:JJ)=A(II,JJ)+S1*CH(I,LI)
   17 RETURN
      END
      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100,100,120
  100 IJ=I*J+1
      NM=N*J+1
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
  130 NM=NM+NI
  140 RETURN
      END
      SUBROUTINE SIMQ(A,B,N,KS)
      DIMENSION A(1),B(1)
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0.
      IT=JJ-J
      DO 30 I=J,N
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
```

```
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

Appendix D

LISTING OF PROGRAM TIM

```
      PROGRAM TIM
      REAL L
      READ 1, NSETS
    1 FORMAT(I10)
      DO 2 MN=1,NSETS
      READ 3, W,S,H,T,L,ER,TAND1,SIGMA
    3 FORMAT(8F10.6)
      TAND1=TAND1*1.0E-04
      SIGMA=SIGMA*1.0E+07
      PRINT 8
    8 FORMAT(1H ,////1H ,*STRIP METALIZATION THICKNESS = 250 MICROINCHES
     1*)
      PRINT 4, W,S,H,T,L,ER,TAND1,SIGMA
    4 FORMAT(1H0,2HW=,F5.1,3X,2HS=,F5.1,3X,2HH=,F5.1,3X,2HT=,F5.1,
     13X,2HL=,F10.6,3X,3HER=,F10.6,3X,3HTAND1=,E10.3,3X,6HSIGMA=,E10.3)
      FACTOR=(1.0E-05)*(1./0.393700)
      W=W*FACTOR
      S=S*FACTOR
      H=H*FACTOR
      T=T*FACTOR
      L=L*FACTOR*(1.E+03)
      E1=ER
      E2=ER+0.01*ER
      CALL TIMF(W,S,H,T,L,E1,SIGMA,C,PDDSQ)
      PDDSQF=PDDSQ
      CALL TIMF(W,S,H,T,L,E2,SIGMA,CP,PDDSQ)
      CALL TIME(W,S,H,T,L,CE)
      A=C*CE
      B=CE/C
      Z0=1./((3.E+08)*SQRT(A))
      V=3.E+08*SQRT(B)
      ARG1=B
      ARG2=CE/CP
      ER1=1./ARG1
      ER2=1./ARG2
      FACTC=10./2.3
      FACTD=27.3/3.E+08
      AC=FACTC*PDDSQF*ER1*Z0
      AD=FACTD*(E1/SQRT(ER1))*((ER2-ER1)/(E2-E1))*TAND1
      PRINT 5
    5 FORMAT(1H0,1X,8HZ0(OHMS),5X,8HV(M/SEC),5X,
     120HALPHAC(DB/M)/SQRT(F),5X,14HALPHAD(DB/M)/F)
      PRINT 6,Z0,V,AC,AD
    6 FORMAT(1H ,2X,F6.2,4X,E10.3,8X,E10.3,11X,E10.3)
    2 CONTINUE
      STOP
      END
      SUBROUTINE TIMF(W,S,H,T,L,ER,SIGMA,CHRG,PDDSQF)
      DIMENSION N(6),X(43,6),Y(43,6),ALPHA(43,6),BETA1(43,6),
     1BETA2(43,6),GAMMA(43,6),CH(43,6),SCH(6),TSCD(43,6,4),A(181,181),
     2A1(181,181)
      REAL L
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
      NO=6
      N(1)=43
      N(2)=41
      N(3)=41
      N(4)=11
```

```
      N(5)=11
      N(6)=34
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      AAA=3.14159*4.E-07*3.14159/SIGMA
      RDSQF=SQRT(AAA)
      FACTN=4.*3.14159*3.14159*RDSQF*8.85E-12
      FACTD=4.*3.14159*1.E-07
      FACTL=FACTN/FACTD
      FACTOR=(1.0E-05)*(1./0.393700)
      DELW=W/20.
      E=ER*E0
      X(1,1)=-W/2.
      Y(1,1)=-0.25*FACTOR
      DO 26 I=1,20
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
   26 Y(IP1,1)=Y(1,1)
      X(22,1)=X(21,1)
      Y(22,1)=0.
      DO 27 I=22,41
      IP1=I+1
      X(IP1,1)=X(I,1)-DELW
   27 Y(IP1,1)=Y(I,1)
      X(43,1)=X(1,1)
      Y(43,1)=Y(1,1)
      DELG=(L/2.-W/2.-S)/20.
      DELT=T/10.
      DELCW2=(W/2.+S)/10.
      X(1,2)=-L/2.
      Y(1,2)=0.
      X(21,2)=-(W/2.)-S
      Y(21,2)=0.
      X(31,2)=X(21,2)
      Y(31,2)=-T
      X(41,2)=0.
      Y(41,2)=-T
      X(1,3)=L/2.
      Y(1,3)=0.
      X(21,3)=(W/2.)+S
      Y(21,3)=0.
      X(31,3)=X(21,3)
      Y(31,3)=-T
      X(41,3)=0.
      Y(41,3)=-T
      DO 28 I=1,19
      IP1=I+1
      X(IP1,2)=X(I,2)+DELG
      Y(IP1,2)=0.
      X(IP1,3)=X(I,3)-DELG
   28 Y(IP1,3)=0.
      DO 228 I=1,9
      IP21=I+21
      IP21M1=IP21-1
      X(IP21,2)=X(IP21M1,2)
      Y(IP21,2)=Y(IP21M1,2)-DELT
      X(IP21,3)=X(IP21M1,3)
      Y(IP21,3)=Y(IP21,2)
      IP31=I+31
      IP31M1=IP31-1
```

```
      X(IP31,2)=X(IP31M1,2)+DELCW2
      Y(IP31,2)=-T
      X(IP31,3)=X(IP31M1,3)-DELCW2
  228 Y(IP31,3)=Y(IP31,2)
      DELS=S/10.
      X(1,4)=-W/2.-S
      Y(1,4)=0.
      X(1,5)=W/2.+S
      Y(1,5)=0.
      DO 29 I=1,10
      IP1=I+1
      X(IP1,4)=X(I,4)+DELS
      Y(IP1,4)=0.
      X(IP1,5)=X(I,5)-DELS
   29 Y(IP1,5)=0.
      DELH=H/2.
      DELL=L/29.
      X(1,6)=-L/2.
      Y(1,6)=0.
      X(32,6)=L/2.
      Y(32,6)=H
      DO 30 I=1,2
      IP1=I+1
      IP32=I+32
      IP32M1=IP32-1
      X(IP1,6)=X(I,6)
      Y(IP1,6)=Y(I,6)+DELH
      X(IP32,6)=X(IP32M1,6)
   30 Y(IP32,6)=Y(IP32M1,6)-DELH
      DO 31 I=1,29
      IP3=I+3
      IP3M1=IP3-1
      X(IP3,6)=X(IP3M1,6)+DELL
   31 Y(IP3,6)=H
      DO 60 I=1,42
      ALPHA(I,1)=1.
      BETA1(I,1)=0.
      BETA2(I,1)=0.
   60 GAMMA(I,1)=1.
      DO 61 I=1,40
      ALPHA(I,2)=1.
      ALPHA(I,3)=1.
      BETA1(I,2)=0.
      BETA1(I,3)=0.
      BETA2(I,2)=0.
      BETA2(I,3)=0.
      GAMMA(I,2)=0.
   61 GAMMA(I,3)=0.
      DO 62 I=1,10
      ALPHA(I,4)=0.
      ALPHA(I,5)=0.
      BETA1(I,4)=E0
      BETA1(I,5)=E
      BETA2(I,4)=E
      BETA2(I,5)=E0
      GAMMA(I,4)=0.
   62 GAMMA(I,5)=0.
      DO 63 I=1,33
      ALPHA(I,6)=0.
```

```
      BETA1(I,6)=E
      BETA2(I,6)=E0
   63 GAMMA(I,6)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=43
      R=1.0E+05
      NAXDIM=181
      NAYDIM=181
      CALL LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DELF=W/20.
      DELE=0.25*FACTOR
      FF1=6.2831852*E*DELF
      FF2=6.2831852*E0*DELE
      FF4=6.2831852*E0*DELF
      CHRG=0.
      DO 50 I=1,20
   50 CHRG=CHRG+FF4*CH(I,1)
      DO 51 I=22,41
   51 CHRG=CHRG+FF1*CH(I,1)
      CHRG=CHRG+FF2*(CH(21,1)+CH(42,1))
      SUM=0.
      DO 64 J=1,NO
      IF(ALPHA(1,J).NE.1.) GO TO 64
      NJ1=N(J)-1
      DO 65 I=1,NJ1
      IP1=I+1
      SEGX=X(IP1,J)-X(I,J)
      SEGY=Y(IP1,J)-Y(I,J)
      SEGARG=SEGX*SEGX+SEGY*SEGY
      SEGLEN=SQRT(SEGARG)
   65 SUM=SUM+CH(I,J)*CH(I,J)*SEGLEN
   64 CONTINUE
      PDDSQF=FACTL*SUM
   54 CONTINUE
      RETURN
      END
      SUBROUTINE LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A(181,181),SCH(NO),B(180),A1(181,181)
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
```

```
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
```

```
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
   16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
   17 RETURN
      END
      SUBROUTINE TIME(W,S,H,T,L,CH1)
      DIMENSION N(3),X(70,3),Y(70,3),ALPHA(70,3),BETA1(70,3),
     1BETA2(70,3),GAMMA(70,3),CH(70,3),SCH(3),TSCD(70,3,4),A(181,181),
     2A1(181,181)
      REAL L
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
      NO=3
      N(1)=41
      N(2)=70
      N(3)=70
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      FACTOR=(1.0E-05)*(1./0.393700)
      DELW=W/40.
      DELG=(L/2.-W/2.-S)/39.
      DELT=T/10.
```

```
      DELCW2=(W/2.+S)/20.
      X(1,1)=-W/2.
      Y(1,1)=0.
      DO 26 I=1,40
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
      Y(IP1,1)=Y(I,1)
      ALPHA(I,1)=1.
      BETA1(I,1)=0.
      BETA2(I,1)=0.
   26 GAMMA(I,1)=1.
      X(1,2)=-L/2.
      Y(1,2)=0.
      X(1,3)=L/2.
      Y(1,3)=0.
      X(40,2)=-W/2.-S
      Y(40,2)=0.
      X(50,2)=X(40,2)
      Y(50,2)=-T
      X(40,3)=W/2.+S
      Y(40,3)=0.
      X(50,3)=X(40,3)
      Y(50,3)=-T
      DO 27 I=1,38
      IP1=I+1
      X(IP1,2)=X(I,2)+DELG
      Y(IP1,2)=Y(I,2)
      X(IP1,3)=X(I,3)-DELG
   27 Y(IP1,3)=Y(I,3)
      DO 227 I=1,9
      IP40=I+40
      IP40M1=IP40-1
      X(IP40,2)=X(IP40M1,2)
      Y(IP40,2)=Y(IP40M1,2)-DELT
      X(IP40,3)=X(IP40M1,3)
  227 Y(IP40,3)=Y(IP40M1,3)-DELT
      DO 228 I=1,20
      IP50=I+50
      IP50M1=IP50-1
      X(IP50,2)=X(IP50M1,2)+DELCW2
      Y(IP50,2)=Y(IP50M1,2)
      X(IP50,3)=X(IP50M1,3)-DELCW2
  228 Y(IP50,3)=Y(IP50M1,3)
      DO 229 I=1,69
      ALPHA(I,2)=1.
      BETA1(I,2)=0.
      BETA2(I,2)=0.
      GAMMA(I,2)=0.
      ALPHA(I,3)=1.
      BETA1(I,3)=0.
      BETA2(I,3)=0.
  229 GAMMA(I,3)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=70
```

```
      R=1.0E+05
      NAXDIM=181
      NAYDIM=181
      CALL LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      CH1=SCH(1)*6.2831852*EU
      RETURN
      END
      SUBROUTINE LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A(181,181),SCH(NO),B(180),A1(181,181)
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
```

```
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
```

```
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
   16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
   17 RETURN
      END
      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100,100,120
  100 IJ=I*J+1
      NM=N*J+1
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
  130 NM=NM+NI
  140 RETURN
      END
      SUBROUTINE SIMQ(A,B,N,KS)
      DIMENSION A(1),B(1)
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0.
      IT=JJ-J
      DO 30 I=J,N
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
```

```
50 A(I1)=A(I1)/BIGA
   SAVE=B(IMAX)
   B(IMAX)=B(J)
   B(J)=SAVE/BIGA
   IF(J-N) 55,70,55
55 IQS=N*(J-1)
   DO 65 IX=JY,N
   IXJ=IQS+IX
   IT=J-IX
   DO 60 JX=JY,N
   IXJX=N*(JX-1)+IX
   JJX=IXJX+IT
60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
65 B(IX)=B(IX)-(B(J)*A(IXJ))
70 NY=N-1
   IT=N*N
   DO 80 J=1,NY
   IA=IT-J
   IB=N-J
   IC=N
   DO 80 K=1,J
   B(IB)=B(IB)-A(IA)*B(IC)
   IA=IA-N
80 IC=IC-1
   RETURN
   END
```

Appendix E

LISTING OF PROGRAM LEMDOC

```
      PROGRAM LEMDOC
      REAL L
      READ 1, NSETS
    1 FORMAT(I10)
      DO 2 MN=1,NSETS
      READ 3,W,S,T,H,L,ER,TAND1,SIGMA
    3 FORMAT(8F10.6)
      TAND1=TAND1*1.E-04
      SIGMA=SIGMA*1.E+07
      PRINT 8
    8 FORMAT(1H ,////1H ,*STRIP METALIZATION THICKNESS = 250 MICROINCHES
     1*)
      PRINT 4,W,S,T,H,L,ER,TAND1,SIGMA
    4 FORMAT(1H0,2HW=,F5.1,3X,2HS=,F5.1,3X,2HT=,
     1F5.1,3X,2HH=,F5.1,3X,2HL=,F10.6,3X,3HER=,F10.6,3X,
     26HTAND1=,E10.3,3X,6HSIGMA=,E10.3)
      FACTOR=(1.0E-05)*(1./0.393700)
      W=W*FACTOR
      S=S*FACTOR
      T=T*FACTOR
      H=H*FACTOR
      L=L*FACTOR*(1.E+03)
      E1=ER
      E2=ER+0.01*ER
      CALL MSCUPF(W,S,T,H,L,E1,SIGMA,CO,CE,PDDSQO,PDDSQE)
      PDDSFO=PDDSQO
      PDDSFE=PDDSQE
      CALL MSCUPF(W,S,T,H,L,E2,SIGMA,COP,CEP,PDDSQO,PDDSQE)
      CALL MSCUPE(W,S,T,H,L,E1,CEO,CEE)
      AE=CE*CEE
      AO=CO*CEO
      BE=CEE/CE
      BO=CEO/CO
      ZOE=1./((3.0E+08)*SQRT(AE))
      ZOO=1./((3.0E+08)*SQRT(AO))
      AA=ZOE*ZOO
      ZO=SQRT(AA)
      VE=3.0E+08*SQRT(BE)
      VO=3.0E+08*SQRT(BO)
      VAVG=(VE+VO)/2.
      RHO=ZOE/ZOO
      C=(RHO-1.)/(RHO+1.)
      CDB=-20.*ALOG10(C)
      ARG1E=BE
      ARG1O=BO
      ARG2E=CEE/CEP
      ARG2O=CEO/COP
      ER1E=1./ARG1E
      ER1O=1./ARG1O
      ER2E=1./ARG2E
      ER2O=1./ARG2O
      FACTC=10./2.3
      FACTD=27.3/3.E+08
      ACE=FACTC*PDDSFE*ER1E*ZOE
      ACO=FACTC*PDDSFO*ER1O*ZOO
      ADE=FACTD*(E1/SQRT(ER1E))*((ER2E-ER1E)/(E2-E1))*TAND1
      ADO=FACTD*(E1/SQRT(ER1O))*((ER2O-ER1O)/(E2-E1))*TAND1
      PRINT 5
    5 FORMAT(1H0,1X,5HC(DB),3X,8HZO(OHMS),2X,9HZOO(OHMS),2X,
```

```
 19HZOE(OHMS),5X,9HVO(M/SEC),5X,9HVE(M/SEC),5X,
 211HVAVG(M/SEC))
  PRINT 6,CDB,ZO,ZOO,ZOE,VO,VE,VAVG
6 FORMAT(1H ,F5.2,5X,F6.2,5X,F6.2,5X,F6.2,5X,E10.3,5X,E10.3,
 15X,E10.3)
  PRINT 7
7 FORMAT(1H0,20HALFACO(DB/M)/SQRT(F),5X,20HALFACE(DB/M)/SQRT(F),5X,
 114HALFADO(DB/M)/F,5X,14HALFADE(DB/M)/F)
  PRINT 8,ACO,ACE,ADO,ADE
8 FORMAT(1H ,5X,E10.3,15X,E10.3,12X,E10.3,9X,E10.3)
2 CONTINUE
  STOP
  END
  SUBROUTINE MSCUPF(W,S,T,H,L,ER,SIGMA,CO,CE,PDDSQO,PDDSQE)
  DIMENSION N(6),X(46,6),Y(46,6),ALPHA(46,6),BETA1(46,6),
 1BETA2(46,6),GAMMA(46,6),CH(46,6),SCH(6),TSCD(46,6,4),A(181,181),
 2A1(132,132)
  REAL L
  COMMON/HELP/A,A1
  EQUIVALENCE(A,A1)
  NO=6
  N(1)=43
  N(2)=43
  N(3)=46
  N(4)=13
  N(5)=13
  N(6)=23
  EO=1./(4.*3.14159*2.99776*2.99776E+09)
  AAA=3.14159*4.E-07*3.14159/SIGMA
  RDSQF=SQRT(AAA)
  FACTN=4.*3.14159*3.14159*RDSQF*8.85E-12
  FACTD=4.*3.14159*1.E-07
  FACTL=FACTN/FACTD
  FACTOR=(1.0E-05)*(1./0.393700)
  DELW=W/20.
  E=ER*EO
  X(1,1)=-(S/2.)-W
  Y(1,1)=H
  X(1,2)=-X(1,1)
  Y(1,2)=H
  DO 26 I=1,20
  IP1=I+1
  X(IP1,1)=X(I,1)+DELW
  Y(IP1,1)=H
  X(IP1,2)=X(I,2)-DELW
26 Y(IP1,2)=H
  X(22,1)=X(21,1)
  Y(22,1)=0.25*FACTOR+H
  X(22,2)=X(21,2)
  Y(22,2)=Y(22,1)
  DO 27 I=22,41
  IP1=I+1
  X(IP1,1)=X(I,1)-DELW
  Y(IP1,1)=Y(I,1)
  X(IP1,2)=X(I,2)+DELW
27 Y(IP1,2)=Y(I,2)
  X(43,1)=X(1,1)
  Y(43,1)=Y(1,1)
  X(43,2)=X(1,2)
```

```
      Y(43,2)=Y(1,2)
      DELL=L/45.
      X(1,3)=-L/2.
      Y(1,3)=0.
      DO 28 I=1,45
      IP1=I+1
      X(IP1,3)=X(I,3)+DELL
   28 Y(IP1,3)=0.
      DELH=H/2.
      X(1,4)=X(1,3)
      Y(1,4)=0.
      X(1,5)=L/2.
      Y(1,5)=0.
      DO 29 I=1,2
      IP1=I+1
      X(IP1,4)=X(1,4)
      Y(IP1,4)=Y(I,4)+DELH
      X(IP1,5)=X(1,5)
   29 Y(IP1,5)=Y(I,5)+DELH
      DEL=(L/2.-(S/2.+W))/10.
      DO 30 I=3,12
      IP1=I+1
      X(IP1,4)=X(I,4)+DEL
      Y(IP1,4)=H
      X(IP1,5)=X(I,5)-DEL
   30 Y(IP1,5)=H
      X(1,6)=-S/2.-W
      Y(1,6)=H+0.25*FACTOR
      X(21,6)=S/2.+W
      Y(21,6)=Y(1,6)+T
      DELT=T/2.
      DO 31 I=1,2
      IP1=I+1
      IP21=I+21
      IP21M1=IP21-1
      X(IP1,6)=X(I,6)
      Y(IP1,6)=Y(I,6)+DELT
      X(IP21,6)=X(IP21M1,6)
   31 Y(IP21,6)=Y(IP21M1,6)-DELT
      DELOV=(2*W+S)/18.
      DO 32 I=3,20
      IP1=I+1
      X(IP1,6)=X(I,6)+DELOV
   32 Y(IP1,6)=Y(I,6)
      DO 47 LLL=1,2
      IF(KKK-0) 38,37,38
C     ODD MODE
   37 CONTINUE
      DO 332 J=1,2
      DO 33 I=1,42
      ALPHA(I,J)=1.
      BETA1(I,J)=0.
      BETA2(I,J)=0.
      GO TO (34,35),J
   34 GAMMA(I,J)=1.
      GO TO 33
   35 GAMMA(I,J)=-1.
   33 CONTINUE
  332 CONTINUE
```

```
      KKK=KKK+1
      GO TO 39
C    EVEN MODE
   38 CONTINUE
      DO 40 J=1,2
      DO 41 I=1,42
      ALPHA(I,J)=1.
      BETA1(I,J)=0.
      BETA2(I,J)=0.
   42 GAMMA(I,J)=1.
   41 CONTINUE
   40 CONTINUE
   39 CONTINUE
      DO 36 I=1,45
      ALPHA(I,3)=1.
      BETA1(I,3)=0.
      BETA2(I,3)=0.
   36 GAMMA(I,3)=0.
      DO 45 I=1,12
      ALPHA(I,4)=0.
      BETA1(I,4)=E
      BETA2(I,4)=E0
      GAMMA(I,4)=0.
      ALPHA(I,5)=0.
      BETA1(I,5)=E0
      BETA2(I,5)=E
   45 GAMMA(I,5)=0.
      DO 46 I=1,22
      ALPHA(I,6)=0.
      BETA1(I,6)=E
      BETA2(I,6)=E0
   46 GAMMA(I,6)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=46
      R=1.0E+05
      NAXDIM=181
      NAYDIM=181
      CALL LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DELF=W/20.
      DELE=0.25*FACTOR
      FF1=6.2831852*E*DELF
      FF2=6.2831852*E*DELE
      FF4=6.2831852*E0*DELE
      CHRG=0.
      DO 50 I=1,20
   50 CHRG=CHRG+FF1*CH(I,1)
      DO 51 I=22,41
   51 CHRG=CHRG+FF1*CH(I,1)
      CHRG=CHRG+(FF2*CH(21,1)+FF4*CH(42,1))
      SUM=0.
      DO 64 J=1,NO
      IF(ALPHA(1,J).NE.1.) GO TO 64
      NJ1=N(J)-1
```

```
      DO 65 I=1,NJ1
      IP1=I+1
      SEGX=X(IP1,J)-X(I,J)
      SEGY=Y(IP1,J)-Y(I,J)
      SEGARG=SEGX*SEGX+SEGY*SEGY
      SEGLEN=SQRT(SEGARG)
   65 SUM=SUM+CH(I,J)*CH(I,J)*SEGLEN
   64 CONTINUE
      PDDSQF=FACTL*SUM
      IF(GAMMA(1,2)-1.) 1001,1000,1001
 1000 PDDSQO=PDDSQF
      CO=CHRG
      GO TO 47
 1001 PDDSQE=PDDSQF
      CE=CHRG
   47 CONTINUE
   54 CONTINUE
      RETURN
      END
      SUBROUTINE LPLACF(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1  TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A(181,181),SCH(NO),B(175),A1(132,132)
      COMMON/HELP/A,A1
      EQUIVALENCE(A,A1)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
```

```
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A,A)
      CALL SIMQ(A,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13

   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A(II,JJ)=0.
```

```
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
   16 A(II,JJ)=A(II,JJ)+S1*CH(I,LI)
   17 RETURN
      END
      SUBROUTINE MSCUPE(W,S,T,H,L,ER,CEO,CEE)
      DIMENSION N(3),X(46,3),Y(46,3),ALPHA(46,3),BETA1(46,3),
     1BETA2(46,3),GAMMA(46,3),CH(46,3),SCH(3),TSCD(46,3,4),A(181,181),
     2A1(132,132)
      COMMON/HELP/A,A1
      EQUIVALENCE (A,A1)
      REAL L
      E0=1./(4.*3.14159*2.99776*2.99776E+09)
      FACTOR=(1.0E-05)*(1./0.393700)
      NO=3
      N(1)=43
      N(2)=43
      N(3)=46
      KKK=0
      DELW=W/20.
      E=ER*E0
      X(1,1)=-(S/2.)-W
      Y(1,1)=H
      X(1,2)=-X(1,1)
      Y(1,2)=H
      DO 26 I=1,20
      IP1=I+1
      X(IP1,1)=X(I,1)+DELW
      Y(IP1,1)=H
      X(IP1,2)=X(I,2)-DELW
   26 Y(IP1,2)=H
      X(22,1)=X(21,1)
      Y(22,1)=0.25*FACTOR+H
      X(22,2)=X(21,2)
      Y(22,2)=Y(22,1)
      DO 27 I=22,41
      IP1=I+1
      X(IP1,1)=X(I,1)-DELW
      Y(IP1,1)=Y(I,1)
      X(IP1,2)=X(I,2)+DELW
   27 Y(IP1,2)=Y(I,2)
      X(43,1)=X(1,1)
      Y(43,1)=Y(1,1)
      X(43,2)=X(1,2)
```

```
      Y(43,2)=Y(1,2)
      DELL=L/45.
      X(1,3)=-L/2.
      Y(1,3)=0.
      DO 28 I=1,45
      IP1=I+1
      X(IP1,3)=X(I,3)+DELL
   28 Y(IP1,3)=0.
      DO 47 LLL=1,2
      IF(KKK-0) 38,37,38
C     ODD MODE
   37 CONTINUE
      DO 332 J=1,2
      DO 33 I=1,42
      ALPHA(I,J)=1.
      BETA1(I,J)=0.
      BETA2(I,J)=0.
      GO TO (34,35),J
   34 GAMMA(I,J)=1.
      GO TO 33
   35 GAMMA(I,J)=-1.
   33 CONTINUE
  332 CONTINUE
      KKK=KKK+1
      GO TO 39
C       EVEN MODE
   38 CONTINUE
      DO 40 J=1,2
      DO 41 I=1,42
      ALPHA(I,J)=1.
      BETA1(I,J)=0.
      BETA2(I,J)=0.
   42 GAMMA(I,J)=1.
   41 CONTINUE
   40 CONTINUE
   39 CONTINUE
      DO 36 I=1,45
      ALPHA(I,3)=1.
      BETA1(I,3)=0.
      BETA2(I,3)=0.
   36 GAMMA(I,3)=0.
      XMIN=0.
      XMAX=0.
      YMIN=0.
      YMAX=0.
      NX=0
      NY=0
      IDIM=46
      R=1.0E+05
      NAXDIM=132
      NAYDIM=132
      CALL LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,TSCD,
     1XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      IF (LLL.EQ.2) GO TO 50
      CEO=2.*3.14159*8.855E-12*SCH(1)
      GO TO 47
   50 CEE=2.*3.14159*8.855E-12*SCH(1)
   47 CONTINUE
      RETURN
```

```
      END
      SUBROUTINE LPLACE(NO,N,X,Y,ALPHA,BETA1,BETA2,GAMMA,CH,SCH,IDIM,R,
     1TSCD,XMIN,XMAX,NX,YMIN,YMAX,NY,NAXDIM,NAYDIM)
      DIMENSION X(IDIM,NO),Y(IDIM,NO),ALPHA(IDIM,NO),BETA1(IDIM,NO),
     1BETA2(IDIM,NO),GAMMA(IDIM,NO),CH(IDIM,NO),TSCD(IDIM,NO,4),N(NO),
     2A1(132,132),SCH(NO),B(129),A(181,181)
      COMMON/HELP/A,A1
      EQUIVALENCE (A,A1)
      PI=3.1415926
      RR=R*R
      DO 1 L=1,NO
      NN=N(L)-1
      DO 1 I=1,NN
      XI=X(I+1,L)-X(I,L)
      YI=Y(I+1,L)-Y(I,L)
      TSCD(I,L,1)=ATAN2(YI,XI)
      TSCD(I,L,2)=SIN(TSCD(I,L,1))
      TSCD(I,L,3)=COS(TSCD(I,L,1))
    1 TSCD(I,L,4)=SQRT(XI*XI+YI*YI)
      JJJ=0
      DO 4 LJ=1,NO
      NJ=N(LJ)-1
      JAJ=JJJ
      JJJ=JJJ+NJ
      DO 4 J=1,NJ
      JJ=JAJ+J
      XJ=(X(J,LJ)+X(J+1,LJ))/2.
      YJ=(Y(J,LJ)+Y(J+1,LJ))/2.
      III=0
      DO 4 LI=1,NO
      NI=N(LI)-1
      III=III+NI
      IAI=III-NI
      DO 4 I=1,NI
      II=IAI+I
      IF(II.EQ.JJ) GO TO 3
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      S1=0.
      S2=0.
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      IF(ALPHA(J,LJ).EQ.0.) GO TO 2
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
    2 TETA1=TSCD(J,LJ,1)-TSCD(I,LI,1)
      S2=0.5*SIN(TETA1)*ALOG(R2/R1)+COS(TETA1)*TETA
      S3=-S2
      GO TO 4
    3 S1=TSCD(I,LI,4)*(1.-ALOG(TSCD(I,LI,4)/2./R))
      S2=-PI
      S3=-PI
    4 A1(JJ,II)=ALPHA(J,LJ)*S1+BETA1(J,LJ)*S2+BETA2(J,LJ)*S3
      M=0
```

```
      DO 5 L=1,NO
    5 M=M+N(L)-1
      JJJ=0
      DO 6 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 6 J=1,NN
      JJ=JAJ+J
    6 B(JJ)=GAMMA(J,L)
      CALL ARRAY(2,M,M,NAXDIM,NAYDIM,A1,A1)
      CALL SIMQ(A1,B,M,KS)
      IF (KS.NE.0) PRINT 100
  100 FORMAT(1H0,18HSYSTEM IS SINGULAR)
      JJJ=0
      DO 7 L=1,NO
      NN=N(L)-1
      JAJ=JJJ
      JJJ=JJJ+NN
      DO 7 J=1,NN
      JJ=JAJ+J
    7 CH(J,L)=B(JJ)
      DO 8 L=1,NO
      NN=N(L)-1
      SCH(L)=0.
      DO 8 I=1,NN
    8 SCH(L)=SCH(L)+TSCD(I,L,4)*CH(I,L)
      IF(NX-1)17,9,10
    9 DX=0.
      GO TO 11
   10 DX=(XMAX-XMIN)/FLOAT(NX-1)
   11 IF(NY-1)17,12,13
   12 DY=0.
      GO TO 14
   13 DY=(YMAX-YMIN)/FLOAT(NY-1)
   14 DO 16 II=1,NX
      XJ=XMIN+FLOAT(II-1)*DX
      DO 16 JJ=1,NY
      YJ=YMIN+FLOAT(JJ-1)*DY
      A1(II,JJ)=0.
      DO 16 LI=1,NO
      NN=N(LI)-1
      DO 16 I=1,NN
      X1=XJ-X(I,LI)
      X2=XJ-X(I+1,LI)
      Y1=YJ-Y(I,LI)
      Y2=YJ-Y(I+1,LI)
      R1=X1*X1+Y1*Y1
      R2=X2*X2+Y2*Y2
      IF((R1.EQ.0.).OR.(R2.EQ.0.)) GO TO 15
      YT=Y(I,LI)*X2-Y(I+1,LI)*X1+YJ*(X(I+1,LI)-X(I,LI))
      XT=X1*X2+Y1*Y2
      TETA=ATAN2(YT,XT)
      S1=TSCD(I,LI,4)*(1.-0.5*ALOG(R2/RR))+0.5*ALOG(R2/R1)*(X1*TSCD(I,LI
     1,3)+Y1*TSCD(I,LI,2))+TETA*(X1*TSCD(I,LI,2)-Y1*TSCD(I,LI,3))
      GO TO 16
   15 S1=TSCD(I,LI,4)*(1.-0.5*ALOG((R1+R2)/RR))
   16 A1(II,JJ)=A1(II,JJ)+S1*CH(I,LI)
   17 RETURN
```

```
      END
      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100,100,120
  100 IJ=I*J+1
      NM=N*J+1
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
  130 NM=NM+NI
  140 RETURN
      END
      SUBROUTINE SIMQ(A,B,N,KS)
      DIMENSION A(1),B(1)
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0.
      IT=JJ-J
      DO 30 I=J,N
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
```

```
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

Appendix F

IMPEDANCE APPROXIMATION FOR LOW-LOSS TRANSMISSION LINES

To show how the characteristic impedance of a low-loss uniform transmission line depends on the loss coefficients, consider the following. The characteristic impedance of such a line, represented in terms of the so-called primary constants R, L, C, and G, is

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{F1}$$

or

$$Z_0 = (Z_0)_{LL} \sqrt{\frac{1 - j\, \dfrac{R}{\omega L}}{1 - j\, \dfrac{G}{\omega C}}} , \tag{F2}$$

where

$$(Z_0)_{LL} = \sqrt{\frac{L}{C}} . \tag{F3}$$

$(Z_0)_{LL}$ is the characteristic impedance of the lossless line with primary constants L and C. Equation (F2) can be approximated by

$$Z_0 \approx (Z_0)_{LL} \left(\frac{1 - j\, \dfrac{R}{2\omega L}}{1 - j\, \dfrac{G}{2\omega C}} \right), \tag{F4}$$

provided $R \ll \omega L$ and $G \ll \omega C$. By simple algebraic manipulation Z_0 in Eq. (F4) can be written as

$$Z_0 \approx (Z_0)_{LL} \left(\frac{1 - j\, \dfrac{\alpha_c}{\beta_{LL}}}{1 - j\, \dfrac{\alpha_d}{\beta_{LL}}} \right), \tag{F5}$$

where

$$\alpha_c = \frac{R}{2} \sqrt{\frac{C}{L}} ,$$

where

$$\alpha_d = \frac{G}{2}\sqrt{\frac{L}{C}}\,, \tag{F7}$$

and

$$\beta_{LL} = \omega\sqrt{LC}\,. \tag{F8}$$

To see the effect of this representation, consider Eq. (5) from the main text:

$$\overline{P}_f = (Re\ Z_0)\ \frac{|V_0|^2}{|Z_0|^2}\ e^{-2\alpha z}\,. \tag{F9}$$

From Eq. (F5), $Re\ Z_0$ can be expressed as

$$Re\ Z_0 \approx (Z_0)_{LL} \left[\frac{1 + \dfrac{\alpha_c \alpha_d}{\beta_{LL}^2}}{1 + \left(\dfrac{\alpha_d}{\beta_{LL}}\right)^2}\right]. \tag{F10}$$

To assess the significance of the bracketed factor, consider the following example using microstrip parameters. Let $\alpha_c = 0.215$, $\alpha_d = 0.132$, and $\beta_{LL} = 52.36$, corresponding to an operating frequency of 1.0 GHz. For this case the bracketed factor in Eq. (F10) has a value of $1.000004 \approx 1.00$. At $f = 100$ GHz the bracketed factor is again approximately 1.00. Hence the approximation

$$Re\ Z_0 \approx (Z_0)_{LL} \tag{F11}$$

is quite adequate for most design purposes. Similarly

$$|Z_0| \approx (Z_0)_{LL}\,. \tag{F12}$$

Appendix G

DERIVATION OF EQ. (23)

The derivation of Eq. (23) given here is similar to that given in Ref. 11. Consider a transmission line composed of N_c conductors and N_D different homogeneous dielectric regions. The total time-averaged energy stored the electric field is given by

$$\overline{W}_e = (1/2) \int_\tau \frac{D^2}{\epsilon} \, d\tau, \tag{G1}$$

where τ is the volume obtained by taking the region enclosed by two cross sections perpendicular to the axis of the tranmsission line, the cross sections being separated by a unit length. The quantities ϵ and D in Eq. (G1) are functions of position.

Consider a slight perturbation of the dielectric constant $\delta\epsilon$. Then

$$\delta\overline{W}_e = \int_\tau \frac{D\,\delta D}{\epsilon} \, d\tau - \int_\tau \frac{D^2\,\delta\epsilon}{2\epsilon^2} \, d\tau. \tag{G2}$$

Equation (G2) can be rearranged using the equations

$$\mathbf{E} = - \nabla\phi, \tag{G3}$$

$$\nabla \cdot \mathbf{D} = \rho, \tag{G4}$$

$$\nabla\cdot(\phi\,\delta\,\mathbf{D}) \equiv - \mathbf{E} \cdot \delta\,\mathbf{D} + \phi\nabla\cdot\delta\mathbf{D}. \tag{G5}$$

Hence Eq. (G2) can be expressed as

$$\delta\overline{W}_e = \int_\tau \phi\,\delta\,\rho \, d\tau - \int_\tau \nabla\cdot(\phi\,\delta\,\mathbf{D}) \, d\tau - 1/2 \int_\tau E^2\,\delta\,\epsilon \, d\tau. \tag{G6}$$

By use of Gauss's theorem

$$\int_{\tau} \nabla \cdot (\phi\,\delta D)\,dV = \sum_{i=1}^{N_c} \phi_i \oint_{C_i} \delta \mathbf{D} \cdot \mathbf{n}\,d\ell = -\sum_{i=1}^{N_c} \phi_i\,\delta Q_i, \tag{G7}$$

where C_i is the contour encircling the ith conductor and $\mathbf{n}$ is a unit vector normal to and directed inward toward the conductor. For zero space charge, $\delta\rho = 0$. Holding the potentials ϕ_i constant and letting $\delta\epsilon$ represent a perturbation of only ϵ_k, which is the dielectric constant of the kth homogeneous dielectric region, the following results:

$$\delta \overline{W}_e = \sum_{i=1}^{N_c} \phi_i\,\delta Q_i - (1/2) \int_{\tau_k} E^2\,\delta\epsilon_k\,d\tau, \tag{G8}$$

where τ_k is the volume enclosed by the kth homogeneous dielectric region. Equation (G8) can be reexpressed as

$$\delta \overline{W}_e = \sum_{i=1}^{N_c} \phi_i\,\delta Q_i - \frac{\delta\epsilon_k}{\epsilon_k}\,\overline{W}_{ek}, \tag{G9}$$

where

$$\overline{W}_{ek} = (1/2) \int_{\tau_k} \epsilon_k E^2\,d\tau. \tag{G10}$$

The summation $\sum_{i=1}^{N_c} \phi_i\,\delta Q_i$ can be rewritten in terms of a coefficient of capacitance as

$$\sum_{i=1}^{N_c} \phi_i\,\delta Q_i = \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} \phi_i \phi_j\,\delta C_{ij} = 2\,\delta \overline{W}_e. \tag{G11}$$

By use of Eqs. (G9) with (G11) the following result is obtained:

$$\frac{\partial \overline{W}_e}{\partial \epsilon_k} = \frac{\overline{W}_{ek}}{\epsilon_k}, \quad k = 1, 2, \ldots, N_D. \tag{G12}$$